B. ARCHITECTURE

MECHANICS OF STRUCTURE – II (AR6301)

BENDING STRESS & SHEAR STRESS – SHEAR STRESS

Lecture - 8

Calculation of Shearing Stresses:

Calculate shear stress and first due to the shear-force acting on a beam the shearing stresses will be set up in the beams. The intensity of shear is given by the formula,

$$q = \frac{FA\bar{y}}{Ib}$$

Where,

F is the shear-force,

A is the area of the beam above the layer considered,

Y is the distance between neutral axis and the center of gravity of area A,

I is the moment of inertia,

B is the width of section.

We should be in a position to compute the maximum shear-force. In case of simply supported beams subjected to different loading condition always the maximum shear will be occurring at the supports and the support reactions will be the value of maximum shear-force. Suppose we have two loads then we need to compute the reactions and whichever reaction is the maximum that will give you the maximum shear-force and standard cases we very well know the value of maximum shear-force. Say if we have a simply supported beam with a central supported load. As I said the maximum shear force will be support reactions, the support reactions at left side will be $W/2$ and the support reactions at right side will be $W/2$ and hence the maximum shear-force will be $W/2$ in this case.

If we have a simply supported beam with non-central load then the reaction at the supports will be Wb/l in the left support and Wa/l in the right support. So based on the A and B values this support reaction will depend or it will be maximum based on the A and B values. In case if you have the simply supported beam subjected to uniformly distributed load over the entire spans. In this case the total load will be $W \times l$ and each support will be taking $Wl/2$ in the reaction. So $Wl/2$ will be the maximum shear force in case of simply supported beam subjected to uniformly distributed load over the entire span.

Similarly if you have a simply supported beam subjected to a triangular load or uniformly varying load then again to find the reactions that is the total load divided by 2 will be the support reaction because of symmetry. So in this case the support reaction will be $2l/4$ on either side. So $2l/4$ will be the maximum shear force. Likewise we should find the maximum shear-force for the given load condition. That's why one need to understand the concept of shear-force and bending moment since that computation will be essential in calculations like shear stresses.

The next one is A, which is the area of the beam above the layer considered. The layer is for example if I am taking rectangular section, I will be interesting in computing the shearing stress at bottom layer, at top layer and at the neutral axis. So like this we have to compute the shearing stresses for each layer. If I considered the layer B then area of the beam above the layer will be the shaded area and then $\bar{y}$ it is equal to the distance between the neutral axis and the center of gravity of area A. Say if you have a rectangle and consider the layer, I am interested in computing the shear stress at this layer. So the area above this layer will be this and $\bar{y}$ stands for the distance between the neutral axis and center of gravity of the area A. In the earlier lectures also we have learned how to compute the moment of inertia for the standard section like rectangle, triangle, hallow circles etc. In case of compound sections also one need to calculate the moment of inertia correctly. For calculating the moment of inertia we need the centroidal distances and we also know how to compute the centroid of any section by splitting the compound section into several sections and at last the b is the width of the beam and the layer considered. In case of rectangle it won't be a problem but in case of T-section or an I-section, if you considered the layer at the top that width is the width of the top flange. Like this, there are width of the flange, web etc. So at the junction we will involve two values of shear stresses by substituting the b value. Two different value for b, we will be getting two different values of shear stress and similarly if I consider this layer width at this level will be width of the web. Now very clear, how to compute the shear stress using this formula,

$$\frac{\mathbf{FA\bar{y}}}{\mathbf{Ib}}$$

Example Problem for Shear Stress = I:

Let us take a example of a wooden beam 100mm wide and 250mm deep and 3m long is carrying a uniformly distributed load of 40kN/m. Determine the maximum shear stress and sketch the variation of shear stress along the depth of the beam. Let the top layer be ‘a’ neutral axis layer be ‘b’ and bottom layer be ‘c’.

Solution:

We have,

$$q = \frac{FA\bar{y}}{Ib}$$

Where, F = shear-force = $wl/2$

=60kN = 60000N.

A = area above the layer ‘a’=0.

Now will have to use this equation where F is the shear-force and we know that maximum shear-force in case of a simply supported beam subjected to uniformly distributed load will be $wl/2$.

$$= \frac{40 \times 3}{2} = \frac{120}{2} = 60kN$$

Hence these 60kN will be the maximum shear-force and we wish to have the value of shear force in Newton as we normally expressed in bending stress. Here A is the area above the layer, say for example let us have the top layer as ‘a’ middle layer as ‘b’ and the bottom layer as ‘c’. At the layer above the axis which is ‘a’ is zero. Then the area at neutral axis ‘b’ will be,

$$F = 60000N$$

$$A = 100 \times 125 = 12500mm^2$$

$$Y = 125 - (125/2) = 62.5mm$$

Where F is the shear-force and A be the area above the layer 'b' and Y be the distance between neutral axis and the centre of gravity of the area A.

$$I = \frac{bd^3}{12}$$

$$= 130.208 \times 10^6 mm^4$$

$$b = 100mm$$

Hence the value of q will be,

$$q = 3.60 N/mm^2.$$

We can also calculate the average shear stress q ave; we know that stress is nothing but force by area. Therefore the average shear stress will be given as,

$$q.ave = \frac{F}{bd} = 2.40 N/mm^2$$

It shall be proved that the maximum shear stress will be 1.5 times averages shear stress and moving on to layer C we have the A which is area above the layer c as,

$$100 \times 250 = 25000 mm^2$$

We have Y which the distance between center of gravity of A and the neutral axis, here the distance y is equal to zero. Hence the shear-force q at layer C is also zero.

So we have the rectangle and we have calculated the stress at varies level. So the stress will be zero at the top and bottom and it will be maximum at neutral axis. The shear stresses variation it will be a parabola. This is the shear stress distribution and the maximum shear stress value will be

$3.60 N/mm^2$ whereas the q average shear stress will be $2.40 \mathrm{N/mm}^2$. So this we should draw the shear stress distribution or we should plot the stress distribution in case of the beam.

Example Problem for Shear Stress = II:

Next example we have an I section which is 200mmx350mm which has a web thickness of 12.5mm and a flange thickness of 25mm. It carries a shearing force of 200kN at a section. Sketch the shear stress distribution.

Solution:

We have a I section with the width of 200mmx350mm and it has the web thickness of 12.5mm and it has the flange thickness of 25mm on top and the bottom flange. It has the shear-force of 200kN at this section and we have to sketch the shear stress distribution. The first step is write the shear stress equation,

$$q = \frac{FA\bar{y}}{Ib}$$

Here first let us find the common terms for all layers. The shear-force is given as,

$$F = 200kN$$

$$= 200 \times 10^3 N$$

The moment of inertia shall be found using the formula,

$$I = \left(\frac{BD^3 - bd^3}{12} \right)$$

We have B is equal to 200mm, D is equal to 350mm and b which will be equal to (200-12.5) and d will be the distance between the flanges or depth of the web. Therefore

$$b = 200 - 12.5 = 187.5mm$$

$$d = 350 - 2 \times 25 = 300mm$$

So substituting the values we will get the moment of inertia as,

$$I = 292.71 \times 10^6 mm^4$$

Let the top layer be 'a' and the bottom layer of top flange be 'b' and the layer at neutral axis be 'c'. We know that the shear stress at top layer is equal to zero. If you compute the stress at a, b and c the same plotted for the portion below neutral axis also where the section being the symmetrical section.

So at the bottom layer of the top flange to calculate the shear stress for flange, we have A which is the area of the flange, Y the distance and b the breath. Therefore the shear stress will be equal to,

$$A = 200 \times 25 = 5000mm^2$$

$$Y = 175 - 25/2 = 162.5mm$$

$$b = 200mm$$

Substituting all the values we get the shear force as,

$$q = 2.78N/mm^2$$

At the bottom layer of top flange q_b for the web of the section there will be only change in the width of the web as 12.5mm. So calculating these we get the shear stress value as 44.41N/mm^2. Here the shear stress is increased due to the reduction in the width and then at neutral axis q_c we have the shear force F, Area A and the moment of inertia I equal to,

$$F = 200 \times 10^3 N$$

$$A_y = 200 \times 25 \times 162.5 + 12.5 \times 150 \times 75 = 953125$$

$$I = 292.71 \times 10^6 mm^4$$

Hence the value at the bottom layer of top flange q_c will be,

$$q_c = 52.1 N / mm^2 .$$

So the shear stress distribution will be zero at the top and bottom. We have two values at the point and we have maximum value at neutral axis. They need to be connected using a parabola. So the shear stress distribution will be a parabola. This is the shear stress distribution across the I section. This is how we should plot the shear stress variations.