B. ARCHITECTURE

MECHANICS OF STRUCTURE – II (AR6301)

BENDING STRESS & SHEAR STRESS – CALCULATION OF BENDING STRESS

Lecture - 7

Example Problem for I Section:

An example of I-section. So we calculating the bending stress for I Section, so this example that we need to compute the bending stresses for the I-section shown in Fig, when it is subjected to a bending moment of 300 N-m. Draw the bending stress distribution. Top flange: 60mmx10mm; Web: 10mmx30mm; Bottom flange: 40mmx10mm.

Solution:

Here the given data are Top flange: 60mm x 10mm; Web: 10mmx30mm; Bottom flange: 40mmx10mm. The problem is to compute the bending stresses for the unsymmetrical I-section. We have the equation,

$$f = \frac{M}{I} y$$

Where, M = bending moment which is equal to 300 N-m = $300 \times 10^3 N - mm.$

We always want to have consistence set of units, so it needs to be N-mm then only we are able to get the bending stress as N-mm^2. Suppose the bending moment value is not given we know how to compute the bending moment for different loading case. So the first case is to compute the maximum bending moment and then we need to compute the moment of inertia. Before calculating I, the center of gravity need to be calculated using the formula,

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$$

$A_1 = 60 \times 10 = 600 mm^2$

Y_1 = distance of c.g of element 1 from top = 10/2=5mm

$A_2 = 10 \times 30$ i.e., width x depth

Y_2 = distance of c.g of element 2 from top = 10+30/2=25mm

$A_3 =$ 40 x 10 = 400 mm^2

$Y_3 =$ 10+30+10/2=45 mm

Once we done this we can easily compute the $\bar{Y}$ value as,

$\bar{Y} = 21.92 mm$ From top

Taking the I-section the first and foremost step is to calculate the c.g of the section from the top and this shall be considered as Y_t.

$Y_t = 21.92 mm$

$Y_b = 50 - Yt = 28.08 mm$

We can easily calculate from the extreme fiber Yt to the bottom fiber Yb. Then the moment of inertia can be calculated using the formula, say we have divided it into three elements so the formula will be,

$$I = \frac{b_1 d_1^3}{12} + b_1 d_1 (\bar{y} - y_1^2) + \frac{b_2 d_2^3}{12} + b_2 d_2 (\bar{y} - y_2^2) + \frac{b_3 d_3^3}{12} + b_3 d_3 (\bar{y} - y_3^2)$$

We know why we are getting the $(\bar{y} - y_1^2)$, because it is the distance between the central axis and the c.g of respective elements. So if I am interested in element 1 then the $(\bar{y} - y_1^2)$ will given me the distance between the neutral axis and the centroid of element 1. Similarly the $(\bar{y} - y_2^2)$ will give the distance between the neutral axis and the centroid of the element 2. So substituting this values we get the I Value as,

$$I = 4.185 \times 10^5 mm^4$$

We have values of bending moment m, we have computed the value of moment of inertia I, and we know the values of $Y_t \,\&\, Y_b$. So substituting these values in the formula we get stress value as follows for Ft will be the bending stress at top fiber and Fb will be the bending stress at the bottom fiber.

$$= M / I \times y_t$$

$$= 15.71 N / mm^2 (compressive)$$

$$= M / I \times y_b$$

$$= 20.13 N / mm^2 (tensile)$$

This is the bending stress distributions across the depth of the beam for the unsymmetrical I section, which have been given.

Example Problem for T Section:

Example is we have given a cantilever beam of span 2.5m has a T section of flange 100mmx25mm and web of size 15mm x 125mm. It is subjected to a point load of 1.6 kN at its free end. Compute the bending stresses we have

$$f = \frac{M}{I} y$$

M = maximum bending moment, Maximum bending moment will occur at the fixed end of a cantilever beam.

$$= W \times I = 1.6 \times 2.5 = 4kN - m = 4 \times 10^6 N - mm$$

In case if we are given a cantilever beam with udl in that case the maximum bending moment will be $\frac{wl^2}{2}$. so whatever may be the given load or different type of beam we should be ready to compute maximum bending moment. Once has to be clear in finding the bending moment as well as we need to be thorough in computing the maximum bending moment and the moment of inertia that means the properties of sections before proceeding to bending stress calculations.

To calculate the moment of inertia first the c.g distance has to be calculated using the formula,

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$$

$$A_1 = 100 \times 25 = 2500 mm^2$$

Y_1 = distance of c.g of element 1 from top = 25/2=12.5mm

$A_2 = 15 \times 125 = 1875 mm^2$ i.e., width x depth

Y_2 = distance of c.g of element 2 from top = 25+125/2=87.5mm

Therefore the c.g from the top is equal to 44.64mm. Hence Y_t = 44.64mm and Y_b = 150-44.64 = 105.36mm.

The next thing to be computed is moment of inertia. So moment of inertia shall be computed using the formula,

$$I = \frac{b_1 d_1^3}{12} + b_1 d_1 (\bar{y} - y_1^2) + \frac{b_2 d_2^3}{12} + b_2 d_2 (\bar{y} - y_2^2)$$

$$= 8.6 \times 10^6 mm^4$$

Hence $f_b = 49 N/mm^2 (compressive)$ & $f_t = 20.76 N/mm^2 (tensile)$,

the stress distribution will be as shown in figure. In case of the cantilever beam the top fiber is subjected to tensile stress and the bottom fiber is subjected to the compressive force. So this is the bending stress distribution across the T section

Composite Beams:

We will see the composite beams because practically we involve the composite materials also. Say for example we have a timber beam and it may be clad with steel. So this will form a composite material.

Composite Beams:

A rectangular section timber beam is 50mm wide and 75mm deep. It is clad with steel plate 10mm thick on the top and bottom. Calculate the maximum stress in the steel and the timber when a moment of 4kN-m is applied. E for timber is 10GPa and for steel is 200GPa.

The section should first be converted into an equivalent steel section with top flange and bottom flange as 50mmx10mm and the web should be of dimension tx75. $t = b \times (Et/Es)$, Where b = width of timber section = 50 x (10/200) = 2.5mm.

Solution:

Let us consider the equivalent steel section top flange of dimension 50mmx10mm and the web section of dimension 2.5mmx75mm. The moment of inertia of the equivalent section shall be found using

$$I = \frac{BD^3 - bd^3}{12}$$

$$B = 50mm, D = 95mm, b = B - t = 47.5mm \ \& \ d = 75mm$$

Hence, $I = 1.9025x10^6 mm^4$

The stress at layer that is at the bottom layer of top flange is given by the formula,

$$= (M / I) \times y_1$$

$$M = 4kN - m = 4 \times 10^6 N - mm$$

$$Y_1 = 37.5mm$$

Hence the value of $f_b = 78.84N / mm^2$.

However the stress in this level will be different because of different E value for timber,

$$fB - timber = fB \times (Et / Es) = 3.942N / mm^2$$

The stress at layer A that is at the top layer of top flange is given by the formula,

$$= (M / I) \times y_2$$

Where Y_2=47.5mm. Hence for fA steel the value is 99.87N/mm^2.

So this how compute for composite section. So far composite section initial transformed sections into equivalent section using the equivalent width computations using the different module of elastic modulus or module of material. Then compute the values, at again we need convert the stress corresponding to the particular material using the formula stress obtained for equivalent section multiplied by young's modulus of the material concern by young's Modulus of the transformed, that will give you the stress at different level for composite sections.