AR6301 : Mechanics of Structures II

Unit 1 –Shear Force and Bending Moment

Lecture -6

Definition of Bending stresses:

First we will see the definition of bending stress. We all know about a bending moment at a section, the bending moment will cause the bending or it will tend to deflect the beam and the internal stresses will be setup which will resist the bending. The process of bending stops when every cross section sets up full resistance to the bending moment. The resistance offered by the internal stresses to the bending is called bending stress.

And we need to make some assumptions in the theory of simple bending. The assumptions made are given below.

- The material of the beam is perfectly homogeneous.
- The beam material is stressed within its elastic limit i.e., the material obeys hook's law.
- The transverse sections which were plane before bending remains plane after bending also
- Each layer of the beam is free to expand or contract independently of the layer above or below it.
- The value of E is same in tension and compression
- The beam is in equilibrium.

These are the assumptions which are essential before proceeding to the theory of simple bending. This is the simple bending equation which we follow for computing bending stresses.

$$\frac{M}{I}=\frac{f}{y}=\frac{E}{R}$$

We will able to compute bending stresses or if the stress is given say for example if we know the limiting bending stress f then we will be in the position what would be the applied load. So this equation will be useful for solving problems of analysis type as well the problems of design type. For example if you limit the bending stress to some value and you know the loading coming across the beam and then with which we can design the cross section of that beam. Say if you know the bending moment or you know the limited bending stress is known then we will be in the position to find what would be the value of moment of inertia to resist the bending moment and to be safer within the allowable bending stress.

Once when the moment of inertia is computed with which we can arrive the cross section of the beam. So let us see the terms involve in the theory of bending equation.

- M – bending moment or moment of resistance
- I – Moment of inertia
- f – bending stress
- y – extreme fiber distance
- E – Young's modulus
- R – radius of curvature

As we say M the bending moment we will be subjected to various loading. So the beam either be the cantilever beam or the beam shall be the simply supported beam or it will be overhanging beam. They may be subjected to any type of loading such as point load, or it may be subjected to uniformly distributed load or we may have loadings in the simply supported beams.

Once we know the loading we will be in a position to compute the bending moment M and we will be interested in calculating the maximum bending moment. For example if we have a cantilever beam with point load at the free end then the maximum bending moment will be W/l. In case of cantilever beam subjected to udl alone then the maximum bending moment will be at the fixed end its value will be equal to $Wl^2/2$.

Similarly if we have a simply supported beam with central point load, the maximum bending moment is $Wl/4$. In case of simply supported beam with respect to uniformly distributed load then the maximum bending moment will be $Wl^2/8$. These are some standard loading cases with which will be in a position to load the values of maximum bending moment. If it is not a standard case then it is a combination then we know how to compute the maximum bending moment which we have studied in unit 1.

So whatever may be the given load said it maybe the cantilever beam or simply supported beam or overhanging beam we should be in a position to calculate the maximum bending moment. So the term M can be computed like that.

Next the term which we have is moment of inertia; we all very well know the moment of inertia of standard sections. For example say if you have rectangle of width b and depth d then the moment of inertia is given by,

$$I = \frac{bd^3}{12}$$

Similarly if you have a circle of diameter d then the moment of inertia is given by,

$$I = \frac{\pi d^3}{64}$$

The units for the measurement of the moment of inertia will be mm^4. The units are very important in case of any computation. Next if you have a hollow circle having external diameter D and internal diameter d, then the moment of inertia is given by,

$$I = \frac{\pi(D^4 - d^4)}{64}$$

If we have a triangle of width b and the height h then the moment of inertia is given by,

$$I = \frac{bh^3}{36}$$

With the formula given we can compute the moment of inertia for the standard section. If it is a compound section say if we have a T-section then the moment of inertia is given by,

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2}$$

In case of compound section we need to find the center of gravity of the compound section. So to find the center of gravity of the compound section we need to take a reference. The reference shall be taken either at the top or at the bottom. So we should find the y1 and y2 values then only we can compute the center of gravity of the compound section from the reference which we have taken that reference shall either be at the top or be at the bottom.

The concept of arriving at the center of gravity and moment of inertia is already been thought in earlier semesters. As I said the area is equal to the area of flange, y_1 is the distance of c.g of flange from top, A_2 be the area of web and y_2 is the distance of c.g of web from top. So we can calculate the value of $\bar{y}$ and then we need to compute the moment of inertia which is required to substitute in the bending equation. Say the moment of inertia of the compound section can be given be,

$$I = I_{self} + Area \times dis\tan ce^2$$

The distance what we mean here is distance between the centroid axis and the c.g of the element one. And then for element two have b_1d_1 the distance between the centroid axis and the c.g of element two.

Then the moment of inertia shall be found using the formula,

$$I = \frac{b_1 d_1^3}{12} + b_1 d_1 (\bar{y} - y_1^2) + \frac{b_2 d_2^3}{12} + b_2 d_2 (\bar{y} - y_2^2)$$

Where b_1 is the width of flange, d_1 is the depth of flange and b_2 is the width of web and d_2 is the depth of web.

In case if we have the I section, for the symmetric I sections i.e., the top flange and the bottom flange have the same dimensions. Then we can easily compute the moment of inertia using the relation,

$$I = \left(\frac{BD^3 - bd^3}{12} \right)$$

Where B = width of flange

D = overall depth of I section

b = B – tw where tw is the width of web

d = D – 2tf, where tf is the thickness of flange

In case if we have a unsymmetrical I sections first we should need to compute the c.g of T-section has to be found using formula,

$$y = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$$

In first case we choose the reference at the top of the 'I' section and we can compute the value for $\bar{y}$ and the c.g of the I section. Also we can choose the reference for the $\bar{y}$ from the bottom of the section. Then the moment of inertia shall be found using the formula,

$$I = \frac{b_1 d_1^3}{12} + b_1 d_1(\bar{y} - y_1^2) + \frac{b_2 d_2^3}{12} + b_2 d_2(\bar{y} - y_2^2) + \frac{b_3 d_3^3}{12} + b_3 d_3(\bar{y} - y_3^2)$$

Using this we can compute the moment of inertia for any section once we are given the I value then the bending equation $\frac{M}{I} = \frac{f}{y}$. We already compute the maximum bending moment for any loading or for any beam.

Where,

M – bending moment or moment of resistance

I – Moment of inertia

f – bending stress

y – extreme fiber distance

E – Young's modulus

R – Radius of curvature

In case of the rectangle the extreme fiber distance for the top fiber is y_t and the extreme fiber distance for the bottom fiber is y_b. So in case of rectangular section,

$$y_b = y_t = \frac{d}{2}$$

And the moment of inertia can be given by the formula,

$$I = \frac{bd^3}{12}$$

For a circle the moment of inertia can be compute using the formula,

$$I = \frac{\pi d^4}{64}$$

Example 1: A rectangular beam 60mm wide and 150mm deep is simply supported over a span of 6m. If the beam is subjected to a central point load of 12kN, find the maximum bending stress induced.

Solution:

Let us start we will draw the cross section first, the cross section of the beam is of width 60mm and the depth is 150mm. And the beam is the simply supported beam and it is subjected to central point load of 12kN. The span is 6m and we have the bending equations,

$$\frac{M}{I} = \frac{f}{y}$$

Hence,

$$f = \frac{M}{I} y$$

The first term will be computed is M which is the maximum bending moment.

$$= WL/4$$

$$= 12 \times 6/4 = 18 kN - m$$

$$= 18 \times 10^6 N - mm$$

We have studied earlier how to compute the maximum bending moment in a simply supported beam. So the actual procedure is to complete the reactions first and then compute the bending moment at the different points. We very well remember the diagram of the simply supported beam as triangle which have the maximum at the center which have the value of WL/4. So the maximum bending moment will be computed using this formula. Since we are interested in computing the bending stress and expressing it in N/mm^2 and here it is given a rectangular section so for the rectangular section the moment of inertia is given by the formula,

$$I = \frac{bd^3}{12}$$

$$I = \frac{60 \times 150^3}{12}$$

$$= 16.875 \times 10^6 mm^4$$

Then we need to compute the bending stress. Let f_b be the bending stress at the bottom of the section which can be calculated using the formula,

$$= (M / I) \times y_b$$

And f_t be the bending stress at the top of the section and it can be calculated using the formula,

$$= (M / I) \times y_t$$

In case of simply supported beams the tensile stress occurs at the bottom and compressive stress occurs at the top. Hence,

$$f_b = 80 N / mm^2 (tensile)$$

$$f_t = 80 N / mm^2 (compressive)$$

The bending stress distribution will be plotted. At the bottom it will be $80N/mm^2$ and at the top it will be $80N/mm^2$. We have used the simple formula M/I = f/y and computed the bending stress at the top fiber as well as at the bottom fiber.

Some problems will be in inverse type that is you will be given the limiting stress f and the cross section and you will be asked to calculate what will be the maximum allowed load in case if it is a simply supported beam or for a cantilever beam. Using this data we can compute the maximum bending moment m. Similarly there may be design problems where we have to compute the width and depth of the beam. These are the possible type of problems which can be asked in the exam in case of bending stress.