AR6301 : Mechanics of Structures II

Unit 1 – Shear Force and Bending Moment

Lecture 5

Uniformly Varying Load

Let us take a cantilever beam subjected to varying load. The load varies from zero at the end B to a intensity of q kN/m at the support A. So we are interested in finding the shear force and the bending moment equation at any section. Let us consider a section xx at a distance x from B. The intensity of uniformly varying load is zero at B and it is q at A. We have to find the intensity at the section xx. So that can be found using linear variation principle. Say for a length of l or span l for the length of l we have the intensity q. So we need to obtain $x = \frac{q}{l}x.$ The intermediate at any length will be $x = \frac{q}{l}x.$ So we can easily develop the Shear force equation and the bending moment equation at the section xx. The Shear force at xx is equal to we need to see the load at the right side of the section xx. In case of uniformly loaded force the Shear force have be given by the area of the loading diagram because the intensity of the load varies linearly and forms like a triangle. So area of the triangle will give you the total load. In case of uniformly distributed we all very well know that the intensity multiplied by length will give you the total load. But in case of uniformly varying load the area of the portion will be shaded as shown in the figure will give the Shear force at xx. So the area of the load will be equal to,

$$= \frac{1}{2} \times x \times \frac{qx}{l}$$

$$= \frac{qx^2}{2l}$$

This will be the shear force value at any section xx. And if we see that equation it is a second order equation. Therefore the shear force diagram will be a parabolic curve for the uniformly varying loaded case. Next we will develop the bending moment case for the uniformly varying load. So we have a section xx and here also we are interested in finding the area of the triangle or the area of the loaded portion and then we are also interested in the c.g distance.

As the intensity we found earlier here also we need the intensity at xx, the intensity is $\frac{2x}{l}$ and the bending moment at any section xx will be equal to load multiplied by the distance. Where load available at the right side of the section is equal to,

$$= \frac{1}{2} \times \frac{qx}{l} \times x$$

In this case we need to identify the center of gravity distance, so this is a triangle and the triangle will have its centroid at this point located at one third of the base. The base length be x, so this will be at a distance of one third of x. Therefore the distance will be one third of x. And the sign convention as usual we have as,

LC	+
LAC	-
RC	-
RAC	+

This moment will be right clockwise and hence the right clockwise will be negative. So this will produce a bending moment of negative bending moment at section xx. This concept we need to follow in case of uniformly varying load and this same principle applies to simply supported beam also, say we have a simply supported beam subjected to a varying load to a triangular load as in some practical cases we will be having a triangular load in a simply supported beam. So in that case we will be in a position to determine the reactions R_A and R_B and then shear force at any section and bending moment at any sections. As a first step in any simply supported beam we need to find the reaction at the supports, so using $\varepsilon_v = 0$ with the sign convention upward positive we will have,

$$R_A + R_B - (\frac{1}{2} \times q \times l)$$

$$R_A + R_B - \frac{1}{2} ql$$

Now we can find $R_A + R_B$ using another value $\varepsilon_m = 0$ say we have a simply supported beam with triangular load like this and we are interested in finding R_A & R_B and sign convention positive. If we take moments about A we will have,

$$\left(\frac{1}{2}\times q\times l\right)\times\frac{l}{2}-R_B\times l=0$$

$$\therefore R_B=\frac{ql}{4}$$

Hence,

$$\therefore R_A=\frac{ql}{4}$$

So in case if we know the value of q is given as numerical value then we can easily substitute and get the value of RA and RB. Once these values are known then we can calculate the shear force and bending moment. Now we will see how to form shear force and bending moment equations. Say we have a simply supported beam subjected to uniformly varying load like this and we have calculated the reactions and this will be $\frac{ql}{4}$. So we need to obtain the shear force equation. So let us take any section xx this is q and we are interested in obtaining the shear force equation at xx. So shear force at xx will be equal to we have a upward reaction of $\frac{ql}{4}$ and we the downward load of $\frac{1}{2}.r$ and the shear force value will be,

$$=\frac{ql}{4}-\frac{1}{2}\times x\times\frac{2qx}{l}$$

Similarly if we want the bending moment we can compute the bending moment equation in the similar way. Say you have the simply supported beam subjected to varying load and we are interested in finding the bending moment equation at xx. So while finding the bending moment equation find the intensity here as we have calculated before it will be,

$$= \frac{2qx}{l}$$

Therefore bending moment at xx will be equal to this reaction and the reaction will be,

$$= \frac{1}{4}ql$$

$$= \frac{ql}{4}.x$$

This will be the moment produced by the reaction at the section xx. And being left clockwise it is being positive. The bending moment of this section at any section will be equal to,

$$= \frac{ql}{4}.x - \left(\frac{1}{2} \times x \times \frac{2qx}{l}\right) \times \frac{1}{3}x$$

So in case of uniformly varying load and the thing which we need to know to find is the area of the triangle and c.g distance will give you the distance which will be useful in finding the bending moment at different points. Now we are in

the position that we can solve any cantilever beam problem subjected to any type of loading let it be point loads or let it be combination of point load and uniformly distributed loads or a cantilever beam subjected to varying load or a simply supported beams subjected to combination of loads.

Now we will move to overhanging beam, so the concept which we have studied in case of cantilever beams and in case of simply supported beams will be helpful in overhanging beam also. So here we will take a simple example say you have a overhanging beam like this subjected to a loads of 5kN and 10kN and with the distance of 2m, 2m and 1m. The overhanging here is 1m. In case of overhanging beam also the procedure remains the same the first step will be calculate support reactions RA and RB. The support reactions can be done as we do earlier using the equation $\varepsilon_v = 0$, that will be

$$R_A + R_B - 5 - 10 = 0$$

$$R_A + R_B = 15$$

And then using the equation $\varepsilon_m = 0$ and the sign convention become positive the equation become 5 x 2 that is we have load of 5kN and load of 10kN so that 5kN will produce a load of 5 x 2 and this will be,

$$5 \times 2 - R_B \times 4 + 10 \times 5$$

$$= 0$$

$$\therefore R_B = \frac{70}{4} = 17.5kN$$

And hence RA will be equal to 15 – 17.5 which will be equal to -2.5kN. So sometime we will also come across negative values for the reactions that indicate that reaction will be downward. That may be happens in case of overhanging beams like this. And in case of overhanging beams say we have calculated the reactions after calculating that we can mark it as downward and upwards with the loads. After calculating the values the shear force and the bending moment computation will become in the similar way. And we can calculate the shear force and bending moment value for the cantilever beams. And the one more point which we need to know in case of overhanging beams we will involve the bending moment diagrams having maximum positive values and negative value at supports. The bending moment value changes from positive to negative and this point we call as contraflexer. This will take place in case of overhanging beams. And this contraflexer may be two points in case of overhanging beams subjected to if you having double overhanging beams then the bending moment diagram in case of double overhanging beam will be like this.

So to obtain the point of contraflexer we need to equate the bending moment at any section to zero. This we do in normal case of finding the maximum bending moment i.e., we know that the maximum bending moment occurs at shear force equal to zero. Similarly if you want to find the point of contraflexer equate bending moment to zero. So this are the subject we learnt in this subject. To review what we have studied calculating shear force and bending moment at cantilever beams and we have studied shear force and bending

moment at simply supported beams. And we have a overview of overhanging beams when in we will involve in point of contraflexer. And the load we looked are point load, several point loads, uniformly distributed loads and uniformly varying loads, similarly combination of loading.

While computing the shear force and the bending moment we have the universal sign conversion as left clockwise, left anticlockwise and right clockwise and right anticlockwise. And for shear force we have left up, left down, right up and right down. We will be in a position to calculate shearing stresses and similarly bending stresses. So shearing stresses and bending stress can be compute when we are clear with finding shear force and bending moment at different sections and the shear force will help in the shear reinforcement. To design the shear reinforcement shear force value is important.

Also we are calculating the deflection of beams which is our next unit. So for deflection we should be in a position that to arrive the maximum bending moment where bending moments is essential in solving deflection problems. And the general deflection equation will be $EI\frac{d^2y}{dx^2} = M_{xx}$.