AR6301: Mechanics of Structures II

Unit 1 –Shear Force and Bending Moment

Lecture -4

Shear-force and Bending moment

In this lecture we will be continuing the shear-force and bending moment concepts and the simply supported beams subjected to several point loads. And also we have combination of load because practically we will be having combination of uniformly distributed load and point loads. Point load will come across especially when we have a lateral beam or a secondary beam supported on a main beam. Secondary beam will transmit load as point load to the main beam. And we will also involve uniformly distributed load, the best example for uniformly distributed load will be the self weight of the beam. Say if you calculate the self weight of a beam we need to know the specific weight of the material of the beam. Once the specific weight of the material of the beam is known multiplied by cross section area will give you the distributed load or self weight of the beam. So we will involve both uniformly distributed load and point load in practical cases. So we need to know how to draw shear force and bending moment and how to calculate the maximum value of the bending moment in order to design the beam subjected to combination of loading. So let us continue the several point load case we have seen in the earlier lecture how to calculate the support reactions in case of a simply supported beam subjected to several point loads. Using the static equations of equilibrium we will be able to calculate the support reactions R_A and R_B and then to calculate shear force draw the simply supported beams mentioning the reaction components and the applied loads.

And as we normally proceed from the left hand to the right hand we will have to consider portion by portion.

So let us consider the portion AC and write the shear-force equation for this portion. If we take the section xx for the portion A_C and if we see to the left side we have a upward load of 12.7kN. So the shear-force at xx is equal to 12.7 kN and since it is acting upward and it is being to the left side and left up is positive so 12.7kN will be the shear-force for this portion A_C.

And next if you move the section to CD as shown in the figure, say consider a section xx for the portion CD and if you look at the left side you an upward reaction of 12.7 and a downward applied load of 10kN. Therefore shear force at xx will be equal to (12.7 – 10kN) which is equal to 2.7kN will be the shear-force for portion CB. Then if you move the portion to DE so imagine we are standing here and looking at the left side of the beam. So first examine what are all the loads available, we have upward load of 12.7kN and a downward load of 10kN and another downward load of 15kN. Therefore shear force at xx is equal to (12.7-10-15) where the positive sign for this 12.7 is left up is positive and the negative sign for the 10 and 15kN is left down being negative. So we will be having the value of -12.3kkN which will be shear force for the portion DE.

The only portion which is leftover is EB, so we need to consider a section in portion EB as shown in figure. If you take the section xx and look at the left side we have a upward force of 12.7kN and a downward load of 10kN, 15kN and 5kN respectively. Therefore the value will be as (12.7-10-15-5) because left being positive and left down being negative this values get there respective sign conventions. The corresponding shear force for the portion EB is equal to -17.3kN. Likewise we will be in a position to calculate the shear force value for different portion if we have number of loads like this.

So this example will help us in solving a problem of simply supported beam subjected to any number of point loads in a similar way as we have proceeded in these examples. Now it is time to plot the shear force diagram for these values and we have the shear force values obtained over varying portions and they shall be plotted like this. For the portion AC we the 12.7kN and we have obtained 2.7kkN for portion CD and for portion CE we have got a negative value of -12.3 and FB we have another negative value of -17.3kN.

If you take this as the base line, the point above the base line will be positive and below the base line will be negative. Therefore we can get the shear force diagram for the simply supported beam subjected to several point loads. The next step is to move towards the bending moment for different sections and different points. And we need write the sign convention initially as we do in all cases. So in case of the bending moment the sign convention will be left clockwise, left anti-clockwise, right clockwise and right anti-clockwise with sign +,-,- and +.

Let us consider the portion AC first and consider a section xx to find the bending moment. So the bending moment at xx will be just consider all the loads available at the left side of the section, so to the left side of the section we have the load of 12.7kN and this 12.7kN will be producing a clockwise moment with respect to the sign convention. So 12.7x will be the bending moment of the section xx for portion AC. And the sign convention is this load will produce a clockwise moment where left clockwise being positive this is +12.7x. Now we will be interested in calculating the bending moment value at different point. So once the equation is ready for portion AC we can easily find the bending moment value at any required point in the portion AC. So accordingly we can generate a tabular column with point, x meters and bending moments in kN-m. At point A x takes the value of 0 and at point C, x takes the value of 2m. Substituting these values in the section we will get the bending moment value as 0 and 25.4kNm.

Next we will move to the portion CD, here if we see to the left side of the section you can have this 12.7 multiplied by x i.e., force into distance and we also have this load of 10kN producing anticlockwise moment. So this 12.7kN will produce a clockwise moment left clockwise being positive and this will produce an anticlockwise moment left anticlockwise being negative. Therefore $12.7 \times x$ will be the moment produced by this force about xx and -10(x-2) i.e., the distance between the point and the section is (x-2) thus we have x up to the section and we have 2m up to C. So the remaining distance will be (x-2). So we will have 12.7x-10(x-2) will be the general equation for bending moment for any point leing in between C and D. So using the tabular column for point C what will be the x value and the bending moment value. Similarly at point D what will be the x and bending moment value in kNm.

Next we should move to portion DE, so bending moment at xx will be equal to we are interested in find the bending moment at this section we have load of 12.7kN, and we have downward load of 10kN and 15kN therefore the bending moment value will be 12.7x-10(x-2) because the distance between point 2 and x will be (x-2) and the distance between D and the section will be (x-3.5) therefore the bending moment equation will be,

$$M_{xx} = 12.7 \times x - 10(x-2) - 15(x-3.5)$$

$$= 12.7x - 10(x-2) - 15(x-3.5)$$

So this will be the general equation and if we substitute the value of x for point D and E then we get the bending moment value in kNm. And there is an alternative way for DE we can also calculate bending moment from the right side. Either we can proceed from the left side or from the right side. And this will be familiar in finding bending moment either from left side or from right side. Because we are very clear with the sign conventions. So if we have that strong basics we can take moment from any side for example we will take bending moment from the right side. So if you consider the section and move to the right side we have this reaction or the value of 17.3kN and we have this load of 5kN. So this 17.3 will produce an anticlockwise moment about the section and let us take x from right side. Then it will be 17.5x-5(x-1). And we will have to substitute the value from the right end so for point D 'x' value will be 0 and for point E 'x' value will be 1m if you substitute these we will get the bending moment in the point D and E. So it is not mandatory to take bending moment only from left side. We can also proceed from the right side. In order to minimize the calculation we can go for finding bending moment from the right end. We will be interested in finding the bending moment at B and E. So at B x takes the value zero and the bending moment value will be zero. And at E x takes the value of 1 and the bending moment value will be 17.3kNm. So if we plot the values it will be zero value at the supports and 25.4kNm at this point C and 29.5kNm at the point D and

17.2kNm at the point B. It will be easier to draw the bending moment diagram or the shear-force if we are familiar with the sign convention. We should be clear in whether the moment produced by any load is clockwise or anticlockwise. Similarly when we are interested in shear-force we should be able to identify whether it acts from the left or right hand or it acts downwards or upwards. This is the example of load combination as we discussed practically we will involve in problems for the combination of load. So here in this example we have the simply supported beam of 1m, 2.5m, 1.5m and 1m. We need to calculate the reactions first. So as we have discussed to calculate the reaction we will have to make use of the equation $\sum v = 0$. With the sign convention upward positive so we have R_A upward and R_B upward and this udl be acting downward and this point load be acting downward. Then we have this downward load so that load intsity is 50 and the distance is 2.5. So (15x2.5) will be the total load and since it acts down it is minus. So -2.65 is the total force given by udl and then we have downward load of 10kN so (-10=0) therefore we get R_A+R_B is equal to total applied downward load of 47.5kN.

Then we can make use of the equation $\sum m = 0$ that is sum of moments about left end is equal to zero or about right end. So we can take moment about the left end. Before that identify what are all the forces available and what will be the moment produced by the forces and if we take moments of all forces about support A the loads which we have are uniformly distributed loads. This downward applied load and upward reaction we have, so if we take moment about A we should find moment produced by the udl A that is moment produced by this 10kN of A and the moment produced by reaction about A.

First let us find the moment produced by the uniformly distributed load about A. So here we have uniformly distributed load of 15kN/m hence the total load acting will be (15x2.5) and the distance from left end will be this 1m plus 2.5 divided by 2. Because we have 1m and the udl acts at the c.g of the length. Therefore it is (1+2.5/2) is the distance about A. And then we have this 10kN. Hence we have,

$$(15\times 2.5)\times\left(1.0+\frac{2.5}{2}\right)+10\times 5-R_B\times 6=0$$

$$\therefore R_B = 22.4kN$$

$$R_A = 47.5-22.4$$

$$=25.1kN$$

So we can find the support reactions always using the static equations of the equilibrium. And next comes the calculation of the shear force, to calculate shear force again we need to proceed section by section. Draw the simply supported beam and mark the support reactions as 25.1 which we have calculated earlier for R_A and R_B with the given applied loads.

Let us consider portion AC if you keep the section at xx for portion AC the shear-force at xx will be equal to we have an upward load of 25.1kN so left up being positive so shear-force at this section will be equal to 25.1kN. And next we need to move the section to CD. So if you move to a section CD then shear-force at xx will be equal to say we have reaction of 25.1kN and we have uniformly distributed load. We have to calculate the uniformly distributed load and we have this distance x and this distance as 1m and therefore the remaining distance will be (x-1). So the shear-force at this section will be,

$$= 25.1 - 15(x-1)$$

So if we are interested in obtaining the shear-force value at varies points we will have to substitute the values of x. So when the section moves to C the 'x' takes the value of 1m and when the section moves to D 'x' takes the value of (1+2.5). So getting a tabular column 'x' in meters and shear-force in kN, substituting the values of different x for point C and D we get bending moment at C and D. Next we need to move the section to DE so shear-force at xx will be equal to the left side of the section we have upward load of 25.1 then minus the full udl is available. Earlier we have considering portion for CD we considered the section in between. Thereby we have the udl contribution as 15(x-1). Now if you see to the section xx we have the full udl of 15 multiplied by 2.5. So this will be the shear-force at xx for portion DE.

Now we will move the section to portion BE then if you considered section in this region. So the shear-force at section xx will be this 25.1kN and then we have downward udl of 15(2.5) and then we have the downward point load of 10kN. Therefore the shear-force at xx for portion BE will be equal to

$$25.1 - 15 \times 2.5 - 10$$

In a similar way we should also write the equations for bending moment. So now we shall write bending moment equation for portion. Let us take the portion AC, so bending moment at xx will be equal to the load acting to the left of xx is 25.1KN and it is acting at a distance of x. So the bending moment at xx will be $25.1\times x$. This will be the equation of bending moment for portion AC. Then if we consider section in CD then bending moment at xx will be equal to 25.1 is the load reaction minus the udl is 15 multiplied by the udl acting over the length of (x-1). And the distance will be (x-1)/2. So we should be clear that 25.1 produce the moment of 25.1x minus the udl acts over the length of (x-1) therefore 15(x-1) and the distance is (x-1)/2. This will be the bending moment equation for portion CD.

Similarly we will have to develop the equations for portion DE and EB. So for portion DE and EB it will be better to develop equation from the right hand side instead of seeing from the left hand side. So let us consider bending moment at any section xx and we will derive equation from the right side as shown in the diagram. So bending moment at xx for portion DE will be equal to,

$$M_{xx} = 22.4\times x - 10\times 1$$

Whereas this 22.4 is multiplied by the distance x, this above equation will be the bending moment value for the portion DE. If you want the bending moment value at point D put x=1m and put x=2.5 meter to get bending moment values at point D and E respectively. Now we will develop equation at xx for portion BE that is the distance is x. So for portion BE the bending moment at xx will be equal to 22.4 multiplied by x. so we have general equation for bending moment and substituting x=0 we get bending moment at B and if we substitute x=1m then we will get the bending moment value at E. So thereby we can obtain bending moment all by points and we can plot the bending moment diagram.