AR6301 : Mechanics of Structures II

Unit 1 –Shear Force and Bending Moment

Lecture -3

Simply Supported Beam

Let us take example of a simply supported beam with central point load. We have loads A and B subjected to central point load subjected to W. Therefore the span is l and the portions l/2 and l/2. As we discussed it is optional to proceed the analysis of shear force and bending moment either from the right side or from left side. In case of cantilever beams we have analyzing from the right end or the free end where we will not involve in the calculation of reactions first directly we can start calculating shear force and bending moment.

However in the case simply supported beams we have support at both the ends so support reactions to be calculated first so to calculate support reactions we will involve the stating equation of equilibrium. For any structure the static equation of equilibrium will be equal to zero.

i) To find reactions:

Using $\sum V = 0[\uparrow (+)]$,

$$R_A + R_B - W = 0$$

$$\therefore R_A + R_B = W --------(i)$$

As mentioned here, the summation of all vertical force is equal to zero and sum of moments is equal to zero. And then we have one more equation called $\sum H = 0$ that is summation of sum of horizontal force is equal to zero. Using the static equation of equilibrium will be equal to able to determine the support reactions again ready to recall what we have learnt about support and reactions.

In case if we have a roller support then there will be only one reaction. In case if we have hinged support we have two reactions as one vertical and another horizontal and in case of fixed support we will have vertical reaction, horizontal reaction and a moment. In this case say we will have a roller support here and another one roller support here so there will be one reaction at A and another reaction at B. So reaction at these two points should be calculated before calculating the shear force and bending moment.

To calculate reaction at A and B let us give there designation as RA and RB. Using $\varepsilon_v = 0$ i.e., sum of all vertical forces is equal to zero with the sign convention upward positive. So if you examine the beam you have RA acting upwards and W acting down and RB acting up. In this equation if you substitute the value $\varepsilon_v = 0$ with the sign convention positive it will be,

$$\varepsilon_v = 0$$

$$R_A + R_B - W = 0$$

$$R_A + R_B = W ------(1)$$

Here the simple logic is the applied load need to be shared and taken care by the support reactions. So that's what has been mentioned in this equation. Then we have $\varepsilon_m = 0$ i.e., sum of moments in that equation is equal to zero for static equilibrium condition. If you want to apply this equation then sign convention need to be used and we will treat clockwise moment as positive and anticlockwise moment as negative.

Now let us take moment of all the support as A with the sign convention clockwise moment as positive. So if you take moments about A we have a load W acting at C and moment produced by C about A is $W \times l/2$ where W is the force and l/2 is the distance. So the moment produced here is,

$$\varepsilon_m = 0$$

$$W \times \frac{l}{2} - R_B \times l = 0$$

$$\therefore R_B = \frac{W}{2}$$

Substituting the above value in equation (1) we get,

$$R_A = W - R_B = \frac{W}{2}$$

$$\therefore R_A = R_B = \frac{W}{2}$$

The simple logic here is it is the symmetrically loaded beam being subjected to central point load. So naturally the supports have to take equally the load. If it is a non-central load then we can't say that the applied load should be equally shared. So after calculating the reactions we can proceed to the shear force analysis. First draw the simply supported beam and mark the reactions which have been found in the first step and don't forget to write the sign convention for the shear force. The sign convention will be Lu, Ld, Ru and Rd.

Let us proceed to the analyzing shear force from the left side and consider the portion AC first and consider the portion xx. To analyze the shear force in section xx you just refer to the left side of the section. To the left side of this section we have the upward load of W/2. So the force acting to the left side of the section will be upward hence it will be positive.

New we can move the section to portion CB. If we have different loads in between it is customaries portion by portion. So in this problem we have portion AB and CB. If you calculate the shear-force for the section xx first let us examine what are all the forces available to the left side of the section. To the left side of this section we have a upward force of W/2 and a downward force of W.

Therefore shear force at this section will be this left up W/2 which is positive and left down which is negative.

So we can plot the shear-force values which we got for the portion AB and CB. And for portion we got the value of $+W/2$ and for portion we have the value of $-W/2$, these values completes the shear-force diagram of simply supported beam subjected to central point load. The shear-force remains constant in the portion udl as in the case of cantilever beam also and it varies linearly in case of distributed load. So this is the shear-force diagram in case of simply supported beam subjected to central point loads. Next we can proceed in calculating the bending moment and for that we need to first write the sign convention of the bending moment.

Consider the portion AC first and consider the section XX at x distance from A. So we are interested in calculating bending moment at section xx. So first examine what are all the loads available to the left side of the section. The only load found in the left side of the section is W/2 and the distance for our section is x. So the bending moment at xx is equal to $W/2 \times x$. This reaction produce a sign convention of clockwise about xx therefore it is positive sign. Then substituting the values at different points say the point at A, x will get the value equal to zero and hence the bending moment also zero and at the point C, x will get the value equal to l/2 with the bending moment value as Wl/4.

Now if you consider the portion CB the bending moment value at xx first examine what are the loads present in the left side of the section. So to the left side of the section we have upward load W/2 and the applied load W. This upward load W/2 will produce a clockwise moment and the applied load produce a anticlockwise moment. So when you are ready with the general equation for bending moment at any section xx, then we can find the bending moment values at different points by substituting the value of x, say namely when this section moves to A, x takes the value of W/2 and when this section moves to B, then x takes the value of l. if you prepare this values in tabular form then the x value at C will be l/2 and the bending moment value will be WL/2 and for the point B the x value will be l and the bending moment is equal to zero.

Next we will move on to a simply supported beam with uniformly distributed load. Here we have a simply supported beam with a span of l subjected to uniformly distributed load W over the entire span and the unit of this will be kN/m. The first and foremost step for this uniformly bending beam is to calculate the support reaction. So to calculate the support reactions we have to use the static equation of equilibrium $\varepsilon_v = 0, \varepsilon_m = 0$ and if necessary $\varepsilon_h = 0$. First let us do $\varepsilon_v = 0$ i.e., sum of all vertical forces with sign convention upward positive. If you notice the diagram the force is acting at A and B will be upward and the uniformly distributed load acting to the downward direction.

$$\varepsilon_v = 0$$

$$R_A + R_B - (W \times l) = 0$$

$$\therefore R_A + R_B = wl - - - - - (1)$$

It is the simply logic that the support has to take care of the applied load being the case to be symmetrically loaded. We can use another form of static equation of equilibrium $\varepsilon_m = 0$ with the sign convention clockwise positive and

we have to take all the moment of force either support A or about support B. So let us take moment of all forces at support A first we will get,

$$\varepsilon_m = 0$$

$$(w \times l) \times \frac{l}{2} - R_B \times l = 0$$

$$\therefore R_B = \frac{wl}{2}$$

Substituting this values in equation (i) we get,

$$R_A = \frac{wl}{2}$$

Now we will move on to shear force at varies point or varies section. So need to remember the sign convention and draw the simply supported beam mark the section xx and mark the reactions $W/2$ on either side of section and also mark the distance from the left end as x. With this if you write an equation for shear force at xx, to the left side we have a load of $W/2$ and left up being positive it is $+W/2$. To the left of section xx we have uniformly distributed loads acting over the length of x therefore $W \times x$ is the load which is acting to the left side of the section xx. If you substitute the value of x you will get the shear-force value. Let us put this in a tabular form as point, distance x and shear-force. At point A the distance x will be zero, so the shear-force will be $Wl/2$. At point B the x value is l and therefore the shear-force value is $-Wl/2$. So the shear-force varies linearly in case of uniformly distributed load.

$$= \frac{wl}{2} - (w \times x)$$

The bending moment for any section will be equal to,

$$= \frac{wl}{2} \times x - (w \times x) \times \frac{x}{2}$$

$$= \frac{wlx}{2} - \frac{wx^2}{2}.$$

If you substitute the value of point A to the point B then you will get the bending moment at different points. Again in a tabular form at point A the value of x will be zero therefore the bending moment value is also zero. At point B the value of x will be l and therefore the bending moment value is zero. The bending moment value will be maximum at a point where the shear-force is zero or at the point where the shear-force changes it sign. So go back to the shear-force diagram and see if you are able to say where the shear-force value is zero or where it changes it sign then at that point if you calculate the bending moment that will be the maximum value of bending moment. We have find where the bending moment value is maximum in case of simply supported beam with uniformly distributed load. Hence equation the shear-force value to zero we get,

$$\frac{wl}{2} - wx = 0$$

$$\therefore x = \frac{l}{2}$$

At $x = \frac{l}{2}$, BM will be maximum and substituting this value of x in bending moment equation we will get maximum value of bending moment as,

$$M_{\max} = \frac{wl \times \frac{l}{2}}{2} - \frac{w\left(\frac{l}{2}\right)^2}{2}$$

$$= \frac{wl^2}{4} - \frac{wl^2}{8}$$

$$M_{\max} = \frac{2wl^2 - wl^2}{8} = \frac{wl^2}{8}$$

Always the support value of reaction will give you the maximum value of shear force and the bending moment will be maximum at the center in case of symmetrically loaded beams and we need to find the point where the bending moment value will be maximum in other cases. And then we will see a simply supported beam supported to several point loads. Say a simply supported beam AB subjected to several point loads of 10kN, 15kN, 5kN with the distance of 2m, 1.5m, 1m, 1m respectively. As we see earlier here also we have to find the support reactions first. So to compute the support reaction we will make use of the static equation of equilibrium $\varepsilon_v = 0$,

$$R_A - 10 - 15 - 5 + R_B = 0$$

$$\therefore R_A + R_B = 30$$

The meaning of this equation is the supports have to take care of the applied loads and need not to be in equal proportion. Now we will take another form of static equation of equilibrium $\varepsilon_m = 0$,

$$10 \times 2 + 15 \times 3.5 + 5 \times 4.5 - R_B \times 5.5 = 0$$

$$\therefore R_B = 17.3kN$$

$$\therefore R_A = 30 - 17.3 = 12.7kN$$