AR6301 : Mechanics of Structures II

Unit 1 –Shear Force and Bending Moment

Lecture 2 – Cantilever beams

Cantilever Beam

Let us take a cantilever beam 6-meter length subjected to a load of 15 kilo newtons at the free end. So if you see in the diagram we have the left support fixed i.e., support A and right point i.e., free end is B and we have a point load of 15 kilo newton acting at the right end. Now we will have to calculate the shear force first and then the bending moment. So whenever you start a problem of analyzing shear force or bending moment write the sign conventions as written here.

To calculate a shear force, consider a section XX which is located at a distance x-m from B. So, I'm interested to calculate the shear force at the given section, at section XX we can see to the left side of the section or to the right side of the section. Let us consider the force acting on the right side of the section XX. To the right side, we have a downward load of 15 kilo newton. Therefore, shear force at section XX will be equal to that 15-kilo newton because to the right of the section we have only that 15 KN force. And the sign convention is + because we have acting load on the right side of our section as per our sign convention right down is positive.

Consider the following example of a cantilever beam, the diagram shows the shear force of a cantilever beam is 15 KN and the shear force will be constant between A to B. The shear force is drawn in the diagram like this we have 15 KN throughout the length. So that is marked as + sign and we have a base line here at the bottom and marked 15 KN above the base line. Now we will calculate bending moment at that section. You go to the section XX, as we define bending moment it is force multiplied by distance. To the right side of the section XX we have a load of 15 KN and the distance between that 15 KN load at your section is Xx. The moment produced at 15 KN in the section Xx will be simply 15 into x and we have the sign minus. Now we will see why the sign is minus.

To the right of section, we have the load 15 KN and the distance is x and the moment produced about the load in Xx section will be clockwise. The sign convention for the bending moment is Lc, LAC, REC and RAC.

Bending Moment:

BM at XX is equal to,

= -15 X x

= -15x

When we consider to the right side of the section, we have a force of 15 kN. Distance between force and section is x. The moment produced by force about xx is Clockwise. Right clockwise (RC is negative). We should be in a position to develop equations like this for shear force and bending moment. And the use of this shear force and bending moment will be you can calculate the bending moment or shear force by varying the distance x at any particular point of interest. Say if we are interested at the free end what will be the bending moment, if I'm interested in calculating bending moment at a point A that is at the left end i.e., at the fixed end what will be the value of the bending moment. By simply substituting the value for x one can able to get the bending moment as well shear force values at any required point.

So, that is clearly mentioned in this tabular column i.e., when the section is at B the value of x will be naturally zero because we are considering the value of x from the right end to the section. So, when the section moves to the point B, x

takes the value of zero. Similarly, when the section is taken to A, x takes the value of l that is length which is 6m in this case. The point B means what is the bending moment at the point B. x takes the value zero hence 15 into zero it is zero. And then at A, x takes the value 6 meters therefore -15 multiplied by 6, the value will be -90 kilo newton. The diagram of bending moment will be like this. Here the negative value can be placed at the below or bottom line and the positive value will be placed at the top line. It will be a linearly varying one from 0 to 90.

Two Point Loads

we will take two loads so that we will be in a position to understand in a better way how to calculate shear force and bending moment at varies points. This is an example of cantilever beam subjected to two loads. One point load is 10 kN acting at point B and the next one is 15 kN acting at 3 meters from B which is named as point C. If you consider the span, i.e., a span is the total length from the support left A to B. It is 6 meters. In earlier case, we had only one load so we considered section in one portion in AB. Here in this case we have two loads hence we need to consider two portions such as portion CB and CA. So, the equation which is developed for any section won't be helpful for the other section. We need to develop equation for every portion.

First, we shall consider a shear force for BC that is we are considering a section XX in portion BC. So, I'm standing at the section XX and viewing to the right side of the section. When you view to the right side you have a downward note of 10 KN. Therefore, shear force at XX is equal to +10. Here the + sign is due to right down. Then when the section is at portion AC like this you have two point loads so one point load is 10 KN and another load is 15 KN. Both are acting down and both are acting to the down side. So, we will have 10+15 that will be the shear force for this portion at XX which is 25 KN. So, for the portion CB the shear force is +10 kN and for the portion CA it is 25 KN.

This will be the resulting shear force diagram. We need to have 10 KN here and the 25 KN there and both are positive. Like this you many have many number

of loads, you consider portion by portion and develop the shear force value and then plot the shear force diagram. We should be clear that what are the force acting at the right side of the section. Similarly, we can also see left side of the section. We can also see from the left side but we are not prefer that because if you proceed from the left end then the support reactions need to be known first before calculating the shear force.

Bending Moment:

Let us first consider the section XX in portion BC at a distance x. So, when I look at the right side of the portion I have a load of 10 KN acting down producing a clockwise moment about the section. Right clock is negative as for our sign convention. Therefore, bending moment at section XX will be -10 multiply by x, hence it is -10x.

Now this equation we call as a general equation for bending moment at XX section and this equation is valid for portion BC only. We can't apply this for the entire section because another load is acting at the other end. So now we are going to substitute values for this equation and find the values for the bending moment. Let us open a tabular column like this, the points and then bending moment values in kilo newton meter. At B the value of x is zero, if we substitute this value in equation then the bending moment value will be zero. And then at point C the x value is 3 meters therefore bending moment will be -10 multiplied with 3. The bending moment will be -30 KN.

Then we can move the section to the portion CA. Consider a section XX at x distance from B. The x is always referred from the right hand. Here we need to calculate the moment produced by the first load in the section and then moment produced by the load in another section. So, the moment produced by 10 KN will be (10 X x = 10x). It produces clockwise moment about the section, so right clockwise is negative hence the sign convention will be negative. The moment produced in this section will be -10x.

And then we have another load of 15 KN, line of action of this load will be here and the section is also here, So the distance is x and the next distance is 3 meters. Then the distance we need will be (x-3). The general equation for bending moment at any section for portion AC will be -10 into x and -15 into (x-3). Now we can calculate the values of the bending moment at points C and A.

Imagine the section is at C, here naturally x takes the value of 3. So, substitute the value of 3 in this equation, we will be getting -30 KN as the bending moment value at point C. Then if we go to point A, the x value becomes 6 meters. Substituting the value in the equation we get the bending moment as -105 KN meter. Now we can simply plot the bending moment values obtained at B, C and A. The corresponding diagram will be like this.

Cantilever Beam with Two Point load

Now we will take a case of cantilever beam with two-point load or uniformly distributed load. For example, we take 30 KN and let the span be 6 meters. So, we are interested in finding shear force and bending moment. First let us develop equation for shear force and then we can develop equation for bending moment. Consider a section XX at x meters from B and develop the shear force equations. And we should remember that shear force is force alone no distance comes here. So, if I stand at the section XX and look at the portion right side which has the load of 30 KN per meter acting over a length of x meters.

Here the intensity is 30 KN per meter and the length is x. Therefore, shear force at this portion will be (30 X x) and it acts to the right down where right down is positive. So +30x will be the equation of shear force. Now when you have this equation then you can easily calculate shear force at different points substituting the value of x. First let us take point B, here x takes the value of zero therefore the value of shear force will be zero. When the section is at A, x takes the value of 6 meters therefore the shear force value will be 180 KN.

We will see how to plot the shear force diagram for this cantilever subjected to a uniformly distributed length. Draw the base line first and at B the shear force value is zero and at A we got 180 KN, it is + sign so mark above the base line then connect these two points with +sign. This will be the shear force diagram. Here we can say the shear force line vary linearly between the points. Because the shear force equation which we develop at any section is equal to (30 X x) which is a linear equation. That is why the diagram is also a linear diagram.

Bending Moment

Now coming to the case of bending moment will arrive the bending moment equation and bending moment diagram in case of uniformly distributed load is 30 KN per meter acting over a length of 6 meters. So, consider a section XX and develop bending moment equation at XX. Bending moment at XX will be equal to force into distance. Force acting to the right side of your section is 30 x. This force of 30 x is multiplied by X/2. This will be the value of the bending moment. Now we concern about the sign. Again, consider to the right side of the section you have udl but udl produces clockwise moment about the section. Then the general equation will be $-30x^2/2$. If we simply it will become $-15x^2$. We will arrive the value of bending moment at different points. So, at point B it will be zero because the value of x is zero at B. When it is in A, the x value is 6 so the bending moment will be -15×6^2, therefore the bending moment at A will be -540.

We need to plot these values by placing the points and we need to draw a parabolic curve to connect these values. Why we are connecting this using a parabolic curve is because the equation we got is a parabolic form of equation. In case of uniformly distributed load the diagrams will be as shown here.