B. ARCHITECTURE

MECHANICS OF STRUCTURE – II (AR6301)

LONG COLUMNS & INDETERMINATE BEAMS - INDETERMINATE BEAMS

Lecture - 16

Rankine's formula:

Rankine's has evaluated formula for computing the load that can be carried by a column as,

$$P = \frac{\sigma_C A}{1 + a\left(\frac{l_{eff}}{k}\right)^2}$$

Where σ_C is the crushing stress of the column of the material. So the column may be made up of concrete or composite materials. So the crushing stress of the materials of the column becomes essential to compute the critical load by using the Rankine's formula. Whereas Euler's formula didn't involve in the crushing stress value.

A is cross sectional area of the column. If we are given a rectangle we know the area is equal to b into d. If it is a compound section or if it is the 'I'section then we know how to compute the area of the compound section. Say compound section comprises several rectangles so area of each rectangle will give you the sum of A.

a is Rankine's constant i.e., $\left(\frac{\sigma_c}{\pi^2 E}\right)^2$. The Rankine's constant will also given on the problem depends on the materials of the column and if it is not given we can compute using the above formula.

l_{eff} is effective (or) equivalent length of the column.

With this idea we will see a problem based on this concept and one more factor which is left out in this formula is k. Where k is the least radius of gyration.

$$= \sqrt{\frac{I_{xx}}{A}} or \sqrt{\frac{I_{yy}}{A}}$$

Whichever is minimum, Say we know what is moment of inertia if we take a rectangle. So for the rectangle the moment of inertia $I_{xx} = bd^3/12$. If I am going to replace the rectangle by a small strip at a distance from the xx axis then at what distance should I keep the strip so that I will get the same moment of inertia for the strip as well for the rectangle. So this is called the distance or the radius of gyration. In a similar way if a rectangle section can be replaced by a equivalent strip at some distance from yy axis then that we call as radius of gyration with respect to yy-axis. Since we know that area in to square of distance will give moment of inertia by using the perpendicular axis or parallel axis theorem, so we should be familiar in moment of inertia of different sections which we have already studied in several units or topics.

Now we will see the problem which is comparing the crippling load by Euler's problem as well as the rankine's formula, so that we will have the idea of both the formula.

Example:

Determine the Euler's crippling load for a hollow cylindrical steel column of 38mm external diameter and 2.5mm thick. Take length of column as 2.3m and hinged at both ends. Take E=205Gpa. Also determine crippling load by rankine's formula using constants as 335 Mpa and 1/7500.

Solution:

This is the problem we can use both rankine's formula as well as the Euler's formula and we will be comparing the results. First we will calculate by Euler's formula,

Euler's crippling load:

$$P = \frac{\pi^2 EI}{l_{eff}^2}$$

$$E = 205 Gpa$$

$$= 2.05 \times 10^5 N/mm^2$$

$$I = \frac{\pi(D_0{}^4 - D_i{}^4)}{64}$$

$$D_0 = 38mm$$

$$D_i = 38 - 2 \times 2.5 = 33mm$$

$$\therefore I = \frac{\pi(38^4 - 33^4)}{64} = 44117.74mm^4$$

$$l_{eff} = l = 2.3m = 2300mm(both.ends.shinged)$$

$$P = \frac{\pi^2 \times 2.05 \times 10^5 \times 44117.74}{2300^2}$$

$$= 16856.63N = 16.86kN$$

Rankine's crippling load:

$$P = \frac{\sigma_C . A}{1 + a\left(\frac{l_{eff}}{k}\right)^2}$$

Here the $\sigma_C \,\&\, a$ are given in the problem.

$$\sigma_C = 335Mpa = 335N/mm^2$$

$$A = \frac{\pi(D_0^2 - D_i^2)}{4}$$

$$= \frac{\pi(38^2 - 33^2)}{4}$$

$$= 278.68mm^2$$

$$a = \frac{1}{7500}$$

$$l_{eff} = l = 2.3m = 2300mm$$

$$k = \sqrt{\frac{I}{A}} = \sqrt{\frac{44117.74}{278.68}} = 12.58mm$$

$$\therefore P = \frac{335 \times 278.68}{1 + \frac{1}{7500}\left(\frac{2300}{12.58}\right)^2} = 17108N$$

$$= 17.11kN$$

So you can compare this value with the value obtained with Euler's formula and have an idea. With this we come to an end of the theory of long columns and short columns.

Introduction to Indeterminate Beams:

Introduction to indeterminate beams. First we should be in a position to know the determinate structure then only we can able to define indeterminacy. We normally use the basic static equations of equilibrium. So the three static equilibriums are $\sum V = 0, \sum H = 0 \;\&\; \sum M = 0$. When these static equations of equilibrium are sufficient to determine the unknown reaction or theunknown forces of a problem then we say the structure as the determinate one and examples of determinate structures will be a cantilever beam, a simply supported beam say this end will be on hinged and this end will be one roller and we have the overhanging beams. In all these cases we can easily find the support reactions using the basic equations of equilibrium. So when we are able to identify the reactions with the help of conditions of equilibrium then we say the structure as the statically determinate one.

Now we will see the indeterminate structure. So in contrary to these indeterminate structures that is the static equations are equilibrium alone and will not be sufficient to calculate the support reactions. Say for example in the case of the propped cantilever beam i.e., a cantilever beam which is propped at the free end. So far we have seen only cantilever beams. Now say this is A and this is B and we have seen the fixed support will offer three reactions that is one vertical reaction, one horizontal reaction and a moment. Whereas in this support we have only R_B. So the equation are the unknown forces in this problem are R_B,V,H and M. There are four unknowns in the problem and we have in hand only three equations of equilibrium or three conditions of equilibrium. Therefore we have one reaction in excess to be found. This we call as the degree of redundancy d_R. If I want to find the degree of redundancy that is equal to,

$$DR = NR - CE$$

That is number of reactions minus the conditions of equilibrium. Hence the number of reaction we have are 4 and the conditions of equilibrium are 3. Say we have a propped cantilever say this is A and this is B. There are four reactions and three conditions of equilibrium therefore the reaction is indeterminate and the degree of redundancy is 1. Now we will take the case of fixed beam. There are two ways of determining the degree of indeterminacy of a two way beam. One is without considering any horizontal force then we will have the vertical reactions and the moment of the unknown. Say the unknowns will be V_A and V_B and this will be M_A and M_B. They need not to be same sometimes they will be same or they need not to be same. So there are four unknowns and the conditions of equilibrium neglecting the condition $\sum H = 0$because we are not considering the horizontal reactions. So the conditions of equilibrium will be two therefore 4-2 is equal to 2. Hence the fixed beam will be statically indeterminate to two degrees and then we will take the case of the continuous beams because these are all examples of indeterminate structures. This is the continuous beams say we have V_A, V_B, V_C.

In case of continuous beam the bending moment at A will be zero and bending moment at C will be zero because they are free ends and we will be having a moment at the continuous support so M_B is one unknown. In addition to V_A, V_B, V_C we have M_B as the unknown. So number of unknowns will be four and we have three equations of equilibrium so we have 4 - 3 is equal to 1.So the degree of indeterminacy here it is 1. So how we will be solving the excess case that is in case of indeterminate beams we should know the concept only based on that concept we will analyze the indeterminacy. Say for example we will start with propped cantilever beamwhich is statically indeterminate to single degree. Say we have a propped cantilever with point A and B. We have R_B so as we discussed the propped cantilever is indeterminate to one degree we will be treating R_B as unknown. So as we discussed we have excess reaction which need to be found using the extra equation. So how do we arrive the extra equation so that the extra equation can be arrived or derived using compatibility condition. So the compatibility condition is nothing but deflection or slope conditions. For example we can use the deflection condition at B. We very well know that if we have the cantilever beam it will deflect like this there will be maximum deflection at the free end and if we provide a support here ultimately it should not allow for this deflection. So using that condition we will be arriving at the value of R_B.

We will see a small example say a method of propped cantilever can be propped at the free end. So first what we should do is remove the prop and we will be analyzing the deflection assuming it as a cantilever beam or we will make it as the determinate structure we will analyze and then remove the load and then apply the redundant force R_B. Now compare the deflection at B when it is subjected to a load here and what will be the deflection at B when it is subjected to the upward load of R_B. So using that we can arrive at the value of unknown R_B. Once R_B is known then the problem will be simple and we can obtain the shear force at any required point and we can obtain the bending moment at any required points and we can also proceed to deflection analysis. So this is the basic concept involved in analyzing the propped cantilever.

Similarly as I said fixed beam is statically indeterminate to two degrees. So in a fixed beam if we are able to calculate the value of end moment M_A and if we are able to calculate the end moment M_B then that become simple. Once we know the values then the other reactions like V_A, V_B can be found using static equations of equilibrium and then the problem can be solved. So to obtain this we will be again using the compatibility condition and the equations which we will be using for fixed beam analysis will be

$$M_A + M_B = \frac{-2A}{l}$$

This will be again derived using the moment area concept which we have studied in the earlier section.

$$M_A + 2M_B = \frac{-6A\bar{x}}{l^2}$$

So with these two additional equations because we have addition unknowns to be found so we use these two additional equations derived from compatibility conditions. So in the fixed beam analysis what they will be doing is first they will be assuming a fixed beam as the simply supported beam subjected to given loading and the bending moment diagram will be drawn. So we are very familiar in calculating the bending moment of a simply supported beam. This A represent the area of the free BMD. Where the BMD is treating the beam as free simply supported beam and A is the area and x bar is the center of gravity distance of the BMD with respect to the left end. So once we know the area and C.G distance then we can easily substitute in these two additional equations and the two additional unknowns can be found and then we have continuous beam so as I said continuous beam of this type have the only additional unknown at the M_B. Once we find this we can find the bending moment and the shearforce. To find this we have several moments such as theorem of three moments, slope deflection method, moment distribution method. So we should have an idea what are the methods that are available for solving the indeterminate beams and in general this methods are classified as two that is one will be force method of analysis and the other will be the displacement method of analysis.

We see the basic concept of these two methods and then that will be conclusion for this unit. In the force method we will be treating the redundant force as the unknown. The strain energy method is an example of force method with which we can solve indeterminate beams and then displacement method of analysis, so as we have discussed the slope deflection method, moment displacement method and we have Cannes method with these methods we can calculate the unknowns. If we treat the rotation of the displacement method as unknown then we will be using the equilibrium equation. So we have an idea of what is an indeterminate method and how to find the degree of indeterminacy and what are the methods available for solving the indeterminate and we do not involve in problems of this type and we also have the latest or recent methods as matrix methods of analysis. So due to the advent or invent of computers we have matrix of analysis of indeterminate beams. The thing which we need to remember in case of finding the degree of indeterminacy is of indeterminate structure. We will have to see for the internal redundancy in case of tresses. If we have the tress and we have numbers. So if the external forces can be solved with the help of the static equilibrium equation and if we have propped cantilever beam and fixed beams and we say that they are externally indeterminate, in case of tresses like this we will have additional internal forces.