B. ARCHITECTURE

MECHANICS OF STRUCTURE – II (AR6301)

COLUMNS – DIRECT & BENDING STRESS

Lecture - 13

Direct Stresses:

The columns are supporting structures in a building. We have seen in the earlier lecture the load distribution. The load distribution takes place from slap and to beams and the beams in terms they transmit to columns. The columns will be of two types that are short columns and long columns.

In case of short columns we will be having crushing stress the phenomenon which will be discussing subsequently and in case of long column will be subjected to buckling. So short column failed by crushing and long columns failed by buckling. We will also see the difference between short column and long column that depends on the slenderness ratio and that also we will discuss while dealing with the theory of long columns. Now we see,

What is a direct stress?

The load passing through the axis of a member is called axial load. If a member is subjected to axial loading, the stress caused will be direct stress, Direct stress $\sigma_d = \frac{load}{area.of.cross.section} = \frac{P}{A}$

Say for example we have the column like this and the supporting column so this is the axis of the column. If our load is concentrated or the load is applied on the axis then we call this as direct stress. So the stress cause by the axial load will be direct stress and we very well know that the stress is given by the load by cross-sectional area. So direct stress if you want to compute the direct stress.

$$\sigma_d = \frac{load}{area.of.cross.section} = \frac{P}{A}$$

On contrary to this we have bending stress say for example if the load on the column does not passes through the axis of the column. But if it is applied from some distance from the axis and we will refer to this distance as eccentricity. So when the load is not concentrated on the axis then we call it as eccentric load. We can see the diagram we can take the column and we have the axis of the column say let as imagine the load applied somewhere here. So for analyzing problem of this type or cases of this types. We introducing two equal and opposite forces along the axis of the column and which will not affect the original case. However the procedure for applying equal and opposite loads is to define the bending stress case. So this we call as eccentricity and the distance between the axis of column and the line of action of the load. Image the case of two equal and opposite load introducing like this. In case of eccentric load we have two equal and opposite axial made to act. The downward load acting of the member will cause direct stress downward load acting at an eccentricity third axial load will form a couple or magnitude P x e, where e is the stress caused by this bending called bending stress. This will cause the direct stress σ_d and the download actually acting downward load and the induced upward along the axis, this two forces will form a couple. So couple we know it is nothing but the moment produced by the two equal and opposite forces. So moment produced will be force multiplied by distance. So the distance which we have between this two parallel forces are given by the eccentricity e. Therefore the moment will be produced when the magnitude of p x e. The bending moment of magnitude of p x e will be caused if we introduced two equal and opposite forces. So the axial load which is acting along the axis downward will cause direct stress and this two forces will be originally applied eccentric load and the upward load acting along the axis they will form the couple of axis which is p x e.

Bending Stresses:

Whenever you have bending moment then we will have bending stress. We will be having the direct stress and the bending stress.

So how to calculate the value of bending stress, we have already studied in earlier chapters that bending stress is given by say we have our basic equation,

$$\frac{M}{I} = \frac{F}{y}$$

We know that bending stress,

$$\sigma_b = \frac{M_y}{I}$$

$M = bending.moment = p \times e$

$y = extreme.fiber.dis\tan ce = b/2$

The extreme fiber column will be like this say if you have a column. The axis of the x and y axis of the column and let the load be applied here and this is our xx axis and this is yy axis. So the load is eccentric with respect to y by axis. We need to find the extreme fiber from the yy axis about which takes place. We will take a case of a column subjected to an eccentric load that eccentricity is with respect to y axis. So we can say the bending takes place about yy axis. Therefore what is the extreme fiber distance with respect to yy axis so that will be equal to this one. So if the width of the column is B then the

$$I = I_{yy} = \frac{db^3}{12}$$

$$\sigma_b = \frac{P \times e \times (b/2)}{(db^3/12)} = \frac{6Pe}{db^2} = \frac{6Pe}{Ab} Since A = bd$$

$I =$ moment of inertia about the axis about which load is eccentric.

We have taken a case of a column subjected to an eccentric load that is eccentricity. So we can say bending takes place about yy axis. Therefore what is the extreme fiber distance with respect to this yy axis. So that will be equal to this. So if the width of the column is B, then this extreme fiber distance will be equal to B/2 as mentioned here. Similarly we want moment of inertia which is also one of the important phenomena in case of bending stress. So which moment of inertia we need to take, because we need to see where the bending moment takes place or about which axis moment the bending moment takes place. So in this diagram bending moment takes place about yy axis.

Therefore we need to compute I_{yy}. So the moment of inertia about yy axis will be,

$$I_{yy} = \frac{db^3}{12}$$

If you want to find out the moment of inertia about xx axis then it is equal to,

$$= \frac{db^3}{12}$$

So we need to first identify where the load is concentrated and the bending moment takes place about which axis that is more important in computing the bending stress. Here we have taken a case where load is acting with respect to yy axis and hence that is equal to,

$$I = I_{yy} = \frac{db^3}{12}$$

Now substituting the values of moment extreme fiber distance and moment of inertia that we get $\frac{6pe}{bd^3}$.

$$\sigma_b = \frac{P \times e \times (b/2)}{(db^3/12)} = \frac{6Pe}{db^2} = \frac{6Pe}{Ab} Since A = bd$$

So $\frac{6Pe}{Ab}$ will be the bending stress.

Example Problem for Direct Stress & Bending Stress = 1:

Now if we have direct stress and bending stress then we have to submit up and if I want to find the total marks we have to submit the sum of the direct and bending stress. We will be required or we will be interested in maximum stress produce similarly we will also be interested in minimum stress produced. The maximum stress will be summation of direct stress and bending stress and the minimum stress will be difference between this two.

Total stress = $\sigma_d \pm \sigma_b$

$$= \frac{P}{A} \pm \frac{6Pe}{Ab}$$

$$= \frac{P}{A}\left(1 \pm \frac{6e}{b}\right)$$

So this will give you the relationship for finding the total stress in a column subjected to eccentric loading. So if we want the maximum stress it is given by $= \frac{P}{A}\left(1 + \frac{6e}{b}\right)$ and if we want the minimum stress then it is given by $= \frac{P}{A}\left(1 - \frac{6e}{b}\right)$ with which we can calculate the maximum and minimum stress in a section. So with this knowledge we can work out a small problem.

Example1:

A rectangular column is 150mm wide and 120mm thick. It carries a load of 180kN at an eccentricity of 10mm in a plane bisecting the thickness. Find the maximum and minimum intensities of stress in the section.

Solution:

Say we have the rectangular column which is 150mm wide and 120mm thick. It carries the load of 180kN at an eccentricity of 10mm in a plane bisecting the thickness. So we need to clearly mark the eccentricity based on the given details. So the eccentricity is 10mm in a given plane bisecting the thickness.

We have two planes, one plane bisects the thickness like this and the other plane will bisect the width like this. So in this problem eccentricity lies in the plane which bisects the thickness. Therefore the load is applied either to the right side or to the left side. However it lies here so this is our yy axis and this is our xx axis. This is the case then directly we can use the formula for maximum stress as,

$$\sigma_{\max} = \frac{P}{A}\left(1 + \frac{6e}{b}\right)$$

$$\sigma_{\min} = \frac{P}{A}\left(1 - \frac{6e}{b}\right)$$

So here P is given as 180kN which is equal to 180 x 1000N then eccentricity is equal to 10mm, area of cross section will be 150mm wide and 120mm thick. Then we need b which is dimensional width which is 150mm. So if you substitute these values we will be getting maximum stress as $14N/mm^2$ and minimum stress as $6N/mm^2$.

$$\sigma_{\max} = 14N/mm^2$$

$$\sigma_{\min} = 6N/mm^2$$

We have the stress distribution normally like this so this is y axis. So this side where the load is concentrated we will have maximum stress and where the load is away we have the minimum stress. So the stress distribution can be drawn like this. In case if the load is applied to the left of yy axis and then this side we will be having the maximum stress and to the opposite side we will have the minimum stress. The stress distribution normally plotted like this. The direct and bending stresses need to be computed based on the given values of load and eccentricity and seeing whether the load produces bending moment value about yy axis or x axis then if it produces about this axis we can directly use the formula $= \frac{P}{A}\left(1 + \frac{6e}{b}\right)$.

Now we will take another example,

Example 2:

A hallow rectangular column is 0.8m x 1.2m and 150mm thick. A vertical load of 2MN is transmitted in the vertical plane bisecting the 1.2m side and at an eccentricity of 100mm from the maximum and minimum stress intensities in the section.

Solution:

This is hallow-rectangular column with width 0.8m and depth 1.2m and 150mm thick on either side. We can keep the width as B and the depth as D and d will be the inner dimension. So the values will be,

$B = 0.8m, D = 1.2m, b = 0.8 - 2\times 0.15 = 0.5m, d = 1.2 - 2\times 0.15 = 0.9m$

Example Problem for Direct Stress & Bending Stress = 2:

So a vertical load of 2MN is transmitted in the vertical plane bisecting the 1.2m side. So the eccentricity is 100mm and the eccentricity lies in the plane which bisects the 1.2m side. So it is need to be marked carefully like this. That is we have the hallow-rectangular column so we will be having two axis like this xx and yy axis. So this axis lies at the 1.2 side and this axis lies yy axis bisects the 0.8m side. But in the given problem it is given that the load acts at an eccentricity lies in the vertical plane bisecting 1.2m side. So the eccentricity will be somewhere here or the load will be applied somewhere here and this is the eccentricity. For problems like this instead of using the formula $= \frac{P}{A}\left(1+\frac{6e}{b}\right)$ or $\frac{P}{A}\left(1-\frac{6e}{b}\right)$. We can use this formula,

$$\sigma_{\max} = \frac{P}{A} + \frac{m}{z}$$

$$\sigma_{\max} = \frac{P}{A} - \frac{m}{z}$$

So it you add the direct and bending stress we will be getting maximum stress and if you find the difference between direct and bending stress we will be getting the minimum stress. Now we will see how the values are calculated. So the moment will be p x e.

$$m = P\times e$$

$$2\times 10^6 \times 100 = 2\times 10^8 N - mm$$

Then we want the Z value, Z is nothing but the section modulus. So when we have moment of inertia and when we know the extreme fiber section I/y normally represents the modulus of eccentricity of a rectangle. This is the section modulus and it will have the units of mm^3 or m^3 or cm^3. Because we have units for moment of inertia as mm^4 and y will be mm therefore the resultant will be mm^3 and we know that I moment of inertia we need to consider I/y because the eccentricity takes place in a plane bisecting the 1.2m side and hence bending moment takes place about yy axis. So I_{yy} for a rectangular section will be,

$$I = \frac{DB^3 - db^3}{12} = 0.04183 m^4$$

If the eccentricity is on the yy axis then it indicates that bending moment takes about xx axis and in that case we need to compute I_{xx}.

$$I_{xx} = \frac{DB^3 - db^3}{12}$$

y is the extreme fiber distance again we need to find the extreme fiber distance with respect to the axis about the bending moment takes place. So the bending moment takes places about y axis as seen in the example one. So y will be equal to,

$$y = B/2 = 0.4m$$

So once we know I and y values we can compute the z value and using this the maximum and minimum stress can also be find and plotted as before.

$$\therefore Z = 0.1046 m^3 = 104.6 \times 10^6 mm^3$$

$$\sigma_{\max} = 5.83 N/mm^2$$

$$\sigma_{\min} = 2.01 N/mm^2$$

So this is the stress distribution diagram for the column.

Eccentric Load for Direct Stress & Bending Stress:

We have seen what is direct stress and bending stress. If there is axial load then direct stress comes into play if there is eccentric load then both direct and bending stress coming to play and we have seen how the direct stress is related in case of eccentric load and how bending stress is produced in case of the eccentric load. Sometimes we will be having columns or the eccentricity about the both axes. Say we have the rectangular column and we will be having yy axis and xx axis. The load may be concentrated here. So in that case say this will be the eccentricity with respect to xx axis e_x and eccentricity with respect to yy axis be e_y. This be all as biaxial bending. So when bending take place about any one axis which is uniaxial bending and when the bending takes place about both the axis that is known as biaxial bending. So in case of columns with eccentric loading about two axis. Then we will be interested in finding stress at the corner. Say it will be corner A, B, C and D. We will have to compute the stress at the four corners of the column so the stress at the corner is given, in that case the bending at xx axis will be there similarly bending about yy axis will be there. We can compute the stress using the formula,

$$\frac{P}{A} \pm \frac{M_{xx}}{Z_{xx}} \pm \frac{M_{yy}}{Z_{yy}}$$

We know that P is the given load and A we can find which is the area of cross section and M_{xx} which is bending moment about x axis. So bending moment about x axis is load multiplied by e_x.

$$M_{xx} = P \times e_x$$

$$Z_{xx} = \frac{I_{xx}}{y}, y = \frac{d}{2}$$

So here how to find the extreme fiber distance, we are interested in Z_{xx} that means bending about x axis it takes place and here what will be the extreme fiber distance. So the extreme fiber distance in this case will be d/2 and bending moment at y axis will be load multiplied by e_y eccentricity with respect to y axis will cause bending moment yy.

$$M_{yy} = P \times e_y$$

$$Z_{yy} = \frac{I_{yy}}{x}, x = \frac{b}{2}$$

Here x means extreme fiber distance, so at what distance the eccentricity is located with respect to extreme fiber that is x. So in this case x will be equal to b/2. Once we know all this value we can compute the values of

$$\frac{P}{A} \pm \frac{M_{xx}}{Z_{xx}} \pm \frac{M_{yy}}{Z_{yy}}.$$

We will be in a position to arrive the stress at the corner. Now we will see a numerical example of this type.

Example:

A masonry pier of size 4m x 3m supports a vertical load of 80kN as shown in fig. Determine the stresses developed at each corner of the pier.

Solution:

Here we have a masonry pier of size 4m x 3m and it supports the vertical load of 80kN with respect to x axis it is given as 0.5 and eccentricity with respect to y axis is given as 1m. The load is concentrated here and we are interested in finding the stress in the corner. So stress at A, B, C and D is required biaxially for this biaxially loaded column. So we know that this 0.5m is e_x that is eccentricity with respect to x axis and this 1m will be e_y because this is the eccentricity with respect to yy axis. Now the stress in corner can be find using the formulas σ_A for stress at corner A, σ_B stress at corner B, σ_C stress at corner C and σ_D stress at corner D. Now we have the direct and bending stress combination,

$$\sigma_A = \frac{P}{A} \pm \frac{M_{xx}}{Z_{xx}} \pm \frac{M_{yy}}{Z_{yy}}$$

We need to see when it will be plus and when it will be minus. For example if you take corner A with respect to x axis the load acts in this side of A acts in the same side then we will have the compressive stress and then plus sign will be used for M_{xx}. Our corner of interest is to the left side of the axis and load acting to the other side.

$$\sigma_A = \frac{P}{A} + \frac{M_{xx}}{Z_{xx}} - \frac{M_{yy}}{Z_{yy}}$$

$$\sigma_B = \frac{P}{A} + \frac{M_{xx}}{Z_{xx}} + \frac{M_{yy}}{Z_{yy}}$$

$$\sigma_C = \frac{P}{A} - \frac{M_{xx}}{Z_{xx}} + \frac{M_{yy}}{Z_{yy}}$$

$$\sigma_D = \frac{P}{A} - \frac{M_{xx}}{Z_{xx}} - \frac{M_{yy}}{Z_{yy}}$$

That is right side of the other axis, so in that case tensile strength will come in play produced by the M_{yy}. So the plus sign and minus sign we need to assign based on the stress which the load passes through the corner. Similarly if we take corner B and it should be noted that always the direct stress is always compressive. So it will always have positive sign only the sign will change for $M_{xx} \& M_{yy}$. Then the corner B is considered we have the load d is acting over the side with respect to x axis and the corner B in the same side.