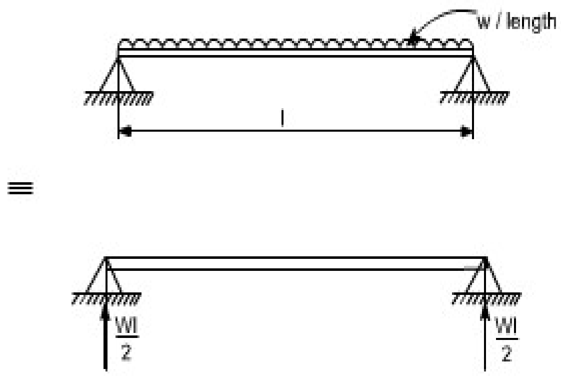
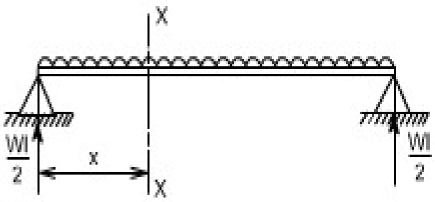
FAQ's

1. Find the slope and deflection of simply supported beam subjected to uniformly distributed load by double integration method?

In this case a simply supported beam is subjected to a uniformly distributed load whose rate of intensity varies as w / length.



In order to write down the expression for bending moment consider any cross-section at distance of x metre from left end support.



$$S.F|_{x-x} = w\left(\frac{l}{2}\right) - w.x$$

$$B.M|_{x-x} = w.\left(\frac{l}{2}\right).x - w.x.\left(\frac{x}{2}\right)$$

$$= \frac{wl.x}{2} - \frac{wx^2}{2}$$

The differential equation which gives the elastic curve for the deflected beam is

$$\frac{d^2y}{dx^2} = \frac{M}{EI} = \frac{1}{EI}\left[\frac{wl.x}{2} - \frac{wx^2}{2}\right]$$

$$\frac{dy}{dx} = \int \frac{wlx}{2EI}dx - \int \frac{wx^2}{2EI}dx + A$$

$$= \frac{wlx^2}{4EI} - \frac{wx^3}{6EI} + A$$

Integrating, once more one gets

$$y = \frac{wlx^3}{12EI} - \frac{wx^4}{24EI} + A.x + B \qquad \text{-----(1)}$$

Boundary conditions which are relevant in this case are that the deflection at each support must be zero.

i.e. at x = 0; y = 0 : at x = l; y = 0

Let us apply these two boundary conditions on equation (1) because the boundary conditions are on y, This yields B = 0.

$$0 = \frac{wl^4}{12EI} - \frac{wl^4}{24EI} + A.l$$

$$A = -\frac{wl^3}{24EI}$$

So the equation which gives the deflection curve is

$$y = \frac{1}{EI}\left[\frac{wLx^3}{12} - \frac{wx^4}{24} - \frac{wL^3x}{24}\right]$$

In this case the maximum deflection will occur at the centre of the beam where x = L/2 i.e. at the position where the load is being applied].So if we substitute the value of x = L/2

Then $$y_{max^m} = \frac{1}{EI}\left[\frac{wL}{12}\left(\frac{L^3}{8}\right) - \frac{w}{24}\left(\frac{L^4}{16}\right) - \frac{wL^3}{24}\left(\frac{L}{2}\right)\right]$$

$$\boxed{y_{max^m} = -\frac{5wL^4}{384EI}}$$

(i) The value of the slope at the position where the deflection is maximum would be zero.

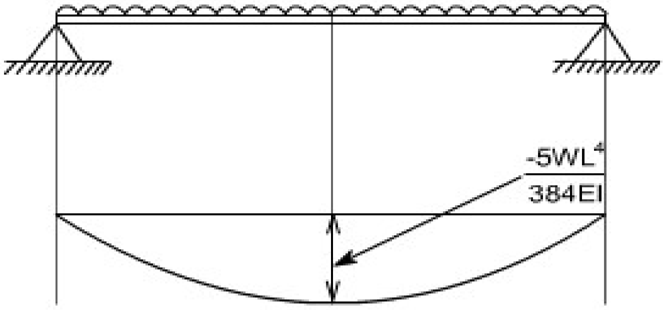
(ii) The value of maximum deflection would be at the centre i.e. at x = L/2.

The final equation which is governs the deflection of the loaded beam in this case is

$$y = \frac{1}{EI}\left[\frac{wLx^3}{12} - \frac{wx^4}{24} - \frac{wL^3x}{24}\right]$$

By successive differentiation one can find the relations for slope, bending moment, shear force and rate of loading.

Deflection (y)

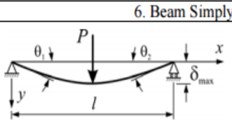
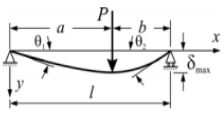
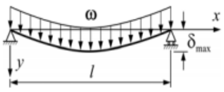
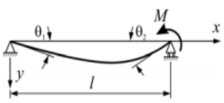
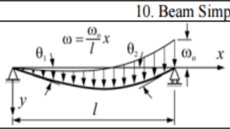


$$yEI = \left[\frac{wLx^3}{12} - \frac{wx^4}{24} - \frac{wL^3x}{24}\right]$$

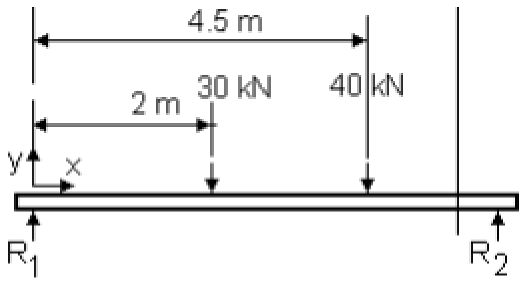
Slope (dy/dx)

$$EI.\frac{dy}{dx} = \left[\frac{3wLx^2}{12} - \frac{4wx^3}{24} - \frac{wL^3}{24}\right]$$

2. Give the expression of slope and deflection of simply supported beam subjected to various loads?

BEAM TYPE	SLOPE AT ENDS	DEFLECTION AT ANY SECTION IN TERMS OF x	MAXIMUM AND CENTER DEFLECTION
6. Beam Simply Supported at Ends – Concentrated load P at the center			
	$\theta_1=\theta_2=\frac{Pl^2}{16EI}$	$y=\frac{Px}{12EI}\left(\frac{3l^2}{4}-x^2\right)$ for $0<x<\frac{l}{2}$	$\delta_{max}=\frac{Pl^3}{48EI}$
7. Beam Simply Supported at Ends – Concentrated load P at any point			
	$\theta_1=\frac{Pb(l^2-b^2)}{6lEI}$ $\theta_2=\frac{Pab(2l-b)}{6lEI}$	$y=\frac{Pbx}{6lEI}(l^2-x^2-b^2)$ for $0<x<a$ $y=\frac{Pb}{6lEI}\left[\frac{l}{b}(x-a)^3+(l^2-b^2)x-x^3\right]$ for $a<x<l$	$\delta_{max}=\frac{Pb(l^2-b^2)^{3/2}}{9\sqrt{3}\,lEI}$ at $x=\sqrt{(l^2-b^2)/3}$ $\delta=\frac{Pb}{48EI}(3l^2-4b^2)$ at the center, if $a>b$
8. Beam Simply Supported at Ends – Uniformly distributed load ω (N/m)			
	$\theta_1=\theta_2=\frac{\omega l^3}{24EI}$	$y=\frac{\omega x}{24EI}(l^3-2lx^2+x^3)$	$\delta_{max}=\frac{5\omega l^4}{384EI}$
9. Beam Simply Supported at Ends – Couple moment M at the right end			
	$\theta_1=\frac{Ml}{6EI}$ $\theta_2=\frac{Ml}{3EI}$	$y=\frac{Mlx}{6EI}\left(1-\frac{x^2}{l^2}\right)$	$\delta_{max}=\frac{Ml^2}{9\sqrt{3}\,EI}$ at $x=\frac{l}{\sqrt{3}}$ $\delta=\frac{Ml^2}{16EI}$ at the center
10. Beam Simply Supported at Ends – Uniformly varying load: Maximum intensity ω_o (N/m)			
	$\theta_1=\frac{7\omega_o l^3}{360EI}$ $\theta_2=\frac{\omega_o l^3}{45EI}$	$y=\frac{\omega_o x}{360lEI}(7l^4-10l^2x^2+3x^4)$	$\delta_{max}=0.00652\frac{\omega_o l^4}{EI}$ at $x=0.519l$ $\delta=0.00651\frac{\omega_o l^4}{EI}$ at the center

3. Determine the slope and deflection of simply supported beam of span 7m and EI value 200MNm^2 given below



First solve the reactions by taking moments about the right end.

$30 \times 5 + 40 \times 2.5 = 7 R_1$ hence $R_1 = 35.71$ kN

$R_2 = 70 - 35.71 = 34.29$ kN

Next write out the bending equation.

$$EI\frac{d^2y}{dx^2} = M = 35710[x] - 30000[x-2] - 40000[x-4.5]$$

Integrate once treating the square bracket as the variable.

$$EI\frac{dy}{dx} = 35710\frac{[x]^2}{2} - 30000\frac{[x-2]^2}{2} - 40000\frac{[x-4.5]^2}{2} + A \ldots\ldots(1)$$

Integrate again

$$EIy = 35710\frac{[x]^3}{6} - 30000\frac{[x-2]^3}{6} - 40000\frac{[x-4.5]^3}{6} + Ax + B \ldots\ldots(2)$$

BOUNDARY CONDITIONS

$x = 0, y = 0$ and $x = 7$ $y = 0$

Using equation 2 and putting $x = 0$ and $y = 0$ we get

$$EI(0) = 35710\frac{[0]^3}{6} - 30000\frac{[0-2]^3}{6} - 40000\frac{[0-4.5]^3}{6} + A(0) + B$$

Ignore any bracket containing a negative value.

$0 = 0 - 0 - 0 + 0 + B$ hence $B = 0$

Using equation 2 again but this time x=7 and y = 0

$$EI(0) = 35710\frac{[7]^3}{6} - 30000\frac{[7-2]^3}{6} - 40000\frac{[7-4.5]^3}{6} + A(7) + 0$$

Evaluate A and $A = -187400$

Now use equations 1 and 2 with x = 3.5 to find the slope and deflection at the middle.

$$EI\frac{dy}{dx} = 35710\frac{[3.5]^2}{2} - 30000\frac{[3.5-2]^2}{2} - 40000\frac{[3.5-4.5]^2}{2} - 187400$$

The last bracket is negative so ignore by puttingin zero

$$200\text{x}10^6\frac{dy}{dx} = 35710\frac{[3.5]^2}{2} - 30000\frac{[3.5-2]^2}{2} - 40000\frac{[0]^2}{2} - 187400$$

$$200\text{x}10^6\frac{dy}{dx} = 218724 - 33750 - 187400 = -2426$$

$\frac{dy}{dx} = \frac{-2426}{200\text{x}10^6} - 0.00001213$ and this is the slopeat the middle.

$$EIy = 35710\frac{[3.5]^3}{6} - 30000\frac{[3.5-2]^3}{6} - 40000\frac{[3.5-4.5]^3}{6} - 187400[3.5]$$

$$200\text{x}10^6 y = 255178 - 16875 - 0 - 655900 = -417598$$

$y = \frac{-417598}{200\text{x}10^6} - 0.00209$ m or 2.09 mm