B. ARCHITECTURE

MECHANICS OF STRUCTURE – II (AR6301)

DEFLECTION OF BEAMS – CANTILEVER BEAM

Lecture - 10

Example Problem for Cantilever Beam:

Let us see an example of a cantilever beam subjected to load partial uniformly distributed load. How we need obtain the slope at required point. So this is an example a cantilever beam of 75mm wide and 200 mm deep is loaded as shown in the figure. Determine slope and deflection at B.E which is equal to 200 Gpa.

Solution:

So a cantilever of beam which is subjected to 75mm of wide and 200mm deep is loaded and we need to find the slope and deflection at the point B. the cross section of the beam is given which is 75mm wide and 200mm deep. The young's modulus is given as 200 Gpa.

$$E = 200 Gpa$$

$$E = 2 \times 10^5 N/mm^2$$

Moment of inertia will be required for the deflection computations and the moment of inertia for a rectangular section is

$$I = \frac{bd^3}{12}$$

Where b is the width which is 75mm and d is the depth of the cross section of the beam which is 200mm.

$$I = \frac{75 \times 200^3}{12}$$

$$I = 50 \times 10^6 mm^4$$

Now we are interested in finding the slope and the deflection at point B. To find the slope at B in case of partial udl like this, we have already seen a partially loaded case like this before and that would be different i.e., that will be loaded at the fixed end and if some portion will be left over. But here the case is different that the load is starts from the free end. So in case if we have partial udl of this type it is customary to introduce an upward load and balanced by a equivalent downward load which will simplify or equal to the process of computing slope and deflection as shown in this diagram. If you see the diagram you can see the span AC, the load which is marked in black is the applied load as 20kN/m. We are introducing a upward load over portion AC similarly it has to be balanced by a downward force to maintain the existing condition. So that will help us to find the slope easily. The slope at B with full udl over AB need to be found first and then it has to be detected with the slope at B due to upward udl over the portion AC, that will give you the net deflection or net value at B. Now we very well know that slope at B due to full udl over AB. It is nothing but the cantilever beam subjected to uniformly distributed load over the entire span. So in that case we have already derived the standard case and the slope will be,

$$= \frac{wl^3}{6EI}$$

Where, $w = 20kN/m = 20N/mm$

$l = 2m = 2000mm$

$EI = 2 \times 10^5 \times 50 \times 10^6$

$EI = 10 \times 10^{12} N - mm^2$

$$(\theta_B) downward.udl = \frac{20 \times 2000^3}{6 \times 10 \times 10^{12}}$$

$= 0.002667$

Then slope at B due to upward udl is given by,

$$= \frac{wl^3}{6EI}$$

Where, $l = 1m(Ac)$

$w = 20N/mm$

$l = 1000mm$

$EI = 10 \times 10^{12} N - mm^2$

$$Slope = \frac{20 \times 1000^3}{6 \times 10 \times 10^{12}} = 0.00033$$

$\therefore \theta B = 0.002667 - 0.00033$

$\theta B = 0.00234 radians$

This type of simplifications we need to take in case if we have partial udl. So the concept which we have studied is we have load up to this hence we want to make use of the standard relation which we have derived. We can use varies concepts for deriving these equation whereas in this case we are we introduce the concept of balancing load i.e., applying a balancing load we can reduce the computation and thereby we can reduced the time of taking in the problem.

In a similar way deflection at B can be computed. So deflection at be will be equal to the same concept that is deflection at B which is y_B is equal to deflection at B due to udl over AB minus deflection at B due to upward udl over AC. First we will find deflection at B due to udl over AB,

$$=\frac{wl^4}{8EI}$$

Where $l = AB = 2m = 2000mm$

$$=\frac{20\times 2000^4}{8\times 10\times 10^{12}}$$

$$=4mm$$

Now we will find the deflection at B due to upward udl over AC i.e., we have the cantilever beam, say we will find deflection at B due to the upward udl of 20N/mm spread over the length of 1m. So here we will use the concept of first we will obtain the deflection at C and then we know the concept that,

$$= y_c + \theta_c \times BC$$

$$y_c = \frac{wl^4}{8EI}$$

Where, $l = 1m = AC = 1000mm$

$$=\frac{20\times 1000^4}{8\times 10\times 10^{12}}$$

$$=0.25mm$$

We already have the slope at C. the slope at C is already compute due to the udl and that multiplied by BC.

$$\theta_c = \frac{wl^3}{6EI}(l = 1m = 1000mm)$$

$$= \frac{20 \times 1000^3}{6 \times 10 \times 10^{12}}$$

$$= 0.0033$$

Therefore deflection at B due to upward udl over AC will given as,

$$= 0.25 + 0.00033 \times 1000$$

$$= 0.58mm$$

Then deflection at B is equal to,

$$= 4 - 0.58$$

$$y_B = 3.42mm$$

Example Problem for Moment Area Method:

Next we will move to the new method called the moment area method. In the moment area method we need to be conversable with drawing the bending moment diagram. If we are conversable with bending moment diagram then the moment area method will be the earliest method for problems governing cantilever beams. So let us take an example of cantilever beam subjected to point load at the free end. We very well know the bending moment at B is equal to zero and bending moment at A will be equal to load into the span and it produces slight clockwise moment so we need to recall our sign convention of bending moment. The bending moment value is plotted as shown in the diagram.

Here we should not commit any mistake in drawing the bending moment diagram since the slope and deflection values which we are going to compute using this moment area method depends on the bending moment values which we have obtained from the diagram and then by moment area method has been proved that as we are interested in finding slope at any point it is given by,

$$Slope.at.B = \frac{Area.of.BMD.upto.B}{EI}$$

$$= \left(\frac{1}{2} \times l \times Wl\right) \div EI$$

$$\theta_B = \frac{Wl^2}{2EI}$$

So it should be remembered that if you want slope at any point find the area of the bending moment diagram up to that point divided by flexural rigidity EI will give you the slope at that point.

Now we will find the deflection at the point, say we have the cantilever with point AB and we got the bending moment diagram like this −Wl. So here if I want deflection at B first I need to find the moment of area of BMD of point up to B. We need to remember the 2/3.l and 1/3.l concept for a triangle. In that case if you get confusion we can remember like this, in case of a triangle there will be two edges one will be the line edge and other will be the point edge. Always the c.g of a triangle lies at the distance of 2/3 of the base length from the point edge.

$$Deflection.at.B = \frac{Moment.of.area.of.BMD.upto.B}{EI}$$

$$= \frac{\left(\frac{1}{2} \times l \times Wl\right) \times \frac{2}{3} l}{EI}$$

$$y_B = \frac{Wl^3}{3EI}$$

Example 2:

The cantilever beam subjected to uniformly distributed load and we very well know that the cantilever beam subjected to udl. We know the slope and deflection values which we have obtained using double integration method. The slope value will be $Wl^3 / 6EI$ at B and the deflection value will be $Wl^4 / 8EI$.

Solution:

The first step in moment area method is draw the free bending moment diagram. We know the bending moment value at B will be equal to zero and bending moment at A will be equal to,

$$M_A = (W \times l) \times \frac{l}{2}$$

$$= -\frac{Wl^2}{2}$$

The negative sign is because the bending moment value has the sign convention of right clockwise which is negative. Now we have drawn the free BMD for the cantilever beam. The next step is we have to compute the area of the bending moment diagram as well the moment of area of bending moment diagram. Say if we have the curve like this then the area of the bending moment diagram will be find by enclosing this diagram with the help of the rectangle. This is the evident that the lower area cover the 2/3 of area of the rectangle and the upper area cover the 1/3 area of the rectangle.

$$Slope.at.B = \frac{Area.of.BMD}{EI}$$

$$= \frac{1}{EI}\left(\frac{1}{3} \times \frac{wl^2}{2} \times l\right)$$

$$\theta_B = \frac{wl^3}{6EI}$$

Now we will move to find the deflection at B, So deflection at B will be moment of area of BMD about B divided by EI. Say we have the c.g distance in case of parabolic curve be ¾ of l. Here again we have a point edge, we have a line edge.

$$= \frac{Moment.of.area.of.BMD.about.B}{EI}$$

$$= \frac{1}{EI}\left[\left(\frac{1}{3} \times \frac{wl^2}{2} \times l\right) \times \left(\frac{3}{4} \times l\right)\right]$$

$$y_B = \frac{wl^4}{8EI}$$

So we have studied this moment area method and the points to remember is if you want slope at any point find the area of the bending moment diagram up to that point and divide by EI and if you want deflection at any point find the moment of area of bending moment diagram up to that point and divide by EI.

Example 3:

A cantilever beam AB of 2m long is carrying a load of 20kN at free end and 30kN at a distance of 1m from the free end. Determine slope and deflection at the free end. Take E = 200 Gpa and $I = 150 \times 10^6 mm^4$

Solution:

Now we have the case of cantilever beam subjected to two point loads of 30kkN and 20kN with the distance of 1m and 1m respectively. As the first step we have to compute the bending moment values at each and every point. So it is very clear that bending moment at B is equal to zero and bending moments at C and A will be equal to,

$$M_B = 0$$

$$M_C = -20 \times 1$$

$$= -20kNm$$

$$M_A = -20 \times 2 - 30 \times 1$$

$$= -70kNm$$

We have the bending moment diagram, in the earlier cases we have a single section but now we have area for this and that can be easily found by dividing like this. So we need to compute the area of the bending moment diagram by spitting it into parts. Here we have area 1, 2 and 3 with 20kNm, 70kNm.

$$A_1 = \frac{1}{2} \times 20 \times 1 = 10$$

$$A_2 = 20 \times 1 = 20$$

$$A_3 = \frac{1}{2} \times 1 \times 50 = 25$$

Therefore total area will be adding these three areas.

$$A = 10 + 20 + 25$$

$$A = \frac{55}{EI}$$

If we have the EI value then we can find the value of θ_B. The θ will be found in terms of radian.