B. ARCHITECTURE

MECHANICS OF STRUCTURES – 1 (AR6201)

PROPERTIES OF SECTION

Lecture – 9

Calculate moment of inertia of a Triangular Section:

Now let us calculate moment of inertia of a triangular section. Let us consider a triangular section ABC Let b be the base width the triangular section and h be the height or altitude of this triangle section, Within this triangular section we shall consider a small strip, let PQ be the that particular strip and the thickness of the strip be dx, dx is the thickness of the strip. Let this strip be at a distance of x from the apex C. By applying similar triangle principle we get

$$\frac{x}{h} = \frac{PQ}{AB} = \frac{PQ}{b}$$

Now we can determine the width of the strip PQ which is

$$PQ = \frac{xb}{h}$$

Area of Strip = $\frac{xb}{h} * dx$

Moment of inertia of this strip about the base AB= $\frac{xb}{h} * dx * (h - x)^2$

Moment of inertia of the whole triangular section strip about the base AB

$$I_{AB} = \int_{x=0}^{x=h} \frac{b}{h} * x(h-x)^2 dx$$

$$I_{AB} = \frac{b}{h} \int_{x=0}^{x=h} \left(xh^3 - x^3 - 2hx^2\right) dx$$

$$I_{AB} = \frac{b}{h}\left[h^2 \frac{x^2}{2} - \frac{x^6}{4} - 2h\frac{x^3}{3}\right]$$

Substitute upper limit x=h and lower limit x=0 so the above equation becomes

$$I_{AB} = \frac{b}{h}\left[\frac{h^4}{2} - \frac{h^4}{4} - 2h\frac{h^3}{3}\right] = \frac{bh^3}{12}$$

$$I_{AB} = \frac{bh^3}{12}$$

Now this is the triangular section which is shown below we have found out the moment of inertia of the triangular section about its base AB, now we need to find this moment of inertia of this triangular section about its centroidal XX axis, so this XX axis passes through the centre of gravity of the triangle and hence it would be called as centroidal XX axis. Distance of the centre of gravity from the base for a triangle will be h/3, we have seen this already where h is the height of the triangle height or the altitude of a triangle. Now by using parallel axis theorem I_{AB} will be

$I_{AB} = I_G + ah^2$

Where I_G is the moment of inertia of rectangular section about its centroidal XX axis which will be I_{xx}. The area of a triangular section and h is the distance between the centroidal XX axis and the line AB which is h/3 here. We know the value of I_{AB} already so the equation becomes

$$\frac{bh^3}{12} = I_{XX} + \frac{1}{2} bh * \left(\frac{h}{3}\right)^2$$

$$\frac{bh^3}{12} = I_{xx} + \frac{bh^3}{18}$$

$$I_{xx} = \frac{bh^3}{12} - \frac{bh^3}{18}$$

By taking LCM and simplifying the equation we get I_{xx} as

$$I_{xx} = \frac{bh^3}{36}$$

So the moment of inertia of a triangular section about its centroidal axis

I_{xx} or I_G is $\frac{bh^3}{36}$

The moment of inertia of a triangular section about the base

$$I_{AB} = \frac{bh^3}{12}$$

Calculate the moment of inertia of Rectangular Section:

Now we shall see a problem for calculating the moment of inertia of rectangular section the dimension of the rectangular section is given length of the rectangular section is 10 cm and breadth of the rectangular section is 4 cm. So the above image is a rectangular section with length 10 cm and breath 4 cm. Calculate the moment of inertia of this rectangular section about its longer and shorter sites about its longer and shorter site. Now this is the longer side and this shorter side. I am extending the longer side, the longer side is named as AB and the shorter side is named as AC. We are asked to calculate the moment of inertia of this rectangular section about AB as well as about AC.

Now this dot is the CG of this rectangular section, this line represents the horizontal centroidal axis of the rectangular section which is XX axis and vertical line passing through this CG will be the centroidal YY axis of this rectangular section.

Moment of inertia of the rectangle about it centroidal XX axis

$$I_{xx} = \frac{b*d^3}{12}$$

Now the dimension parallel to x-axis is AB which will be taken as the breadth b so b here will be equal 10 cm, so the other dimension which is perpendicular to this XX axis other dimension perpendicular XX axis is 4 cm will be the depth here

$$I_{xx} = \frac{10*4^3}{12}$$

Moment of inertia of this rectangular section about its longer side is

$$I_{AB} = I_G + ah^2$$

$$I_{AB} = \frac{10*4^3}{12} + (2)^2$$

$$I_{AB} = 213.33cm^4$$

Similarly we calculate the moment of inertia of this rectangular section about its shorter side which is AC.

$$I_{AC} = I_{yy} + ah^2$$

$$I_{yy} = \frac{4*10^3}{12}$$

$$I_{AC} = \frac{4*10^3}{12} + 40*(5)^2$$

$$I_{AC} = 1333.33cm^4$$

So this the moment of inertia of the rectangular section about the shorter side AC.

Calculate the moment of inertia of the Section:

Calculate the moment of inertia of the section shown below about its centroidal axis XX and YY axis. Now as the first step we have to locate these centre of gravity for this section, locating the centre of gravity of this given section will be done by moment area method which we have discussed already.Now the given section is a unsymmetrical I section, which is a combination of 3 rectangles, now we shall divide this I section into 3 rectangles like this by drawing lines here, so this will be rectangle 1, this will be the 2nd rectangle and this will be the final rectangle which is the numbered as 3. Now the coordinates of the CG of this I section will be computed with reference to axis, the reference axis you see will be the bottom most line, so this is the reference line and the left line, left end line of the figure. Now we shall calculate the value of x bar for this I section, so centre of gravity will be somewhere here, distance of the centre of gravity from the left line is $\bar{x}$ and the distance of the centre of gravity from the bottom line this distance will be $\bar{y}$. Now we are going to calculate the value of $\bar{x}$ and $\bar{y}$.

$$\bar{x} = \frac{a_1x_1 + a_2x_2 + a_3x_3}{a_1 + a_2 + a_3}$$

The area of the first rectangle a_1 = 6*2 =12cm^2 and x_1 is the distance of CG of rectangle 1, so the distance of this CG from this left end line which is named as y and the bottom line is named as x, x_1 = 5cm.

The area of the second rectangle a_2 = 10*2 = 20cm^2 and x_2 is the distance of CG of rectangle 2 which is also x_2 = 5cm.

Next we shall calculate area a_3, the area of the third rectangle a_3 = 10*2 =20cm^2 and x_3 is the distance of CG of rectangle 3 which is also x_2 = 5cm.

So if u do the substitution and do the calculation $\bar{x}$ will be

$$\bar{x} = \frac{12*5+20*5+20*5}{12+20+20} = 5\text{cm}$$

We should also calculate the value of $\bar{y}$ which is

$$\bar{y} = \frac{a_1y_1 + a_2y_2 + a_3y_3}{a_1 + a_2 + a_3}$$

Now we shall calculate the value of y_1, distance of CG of rectangle 1 from the bottom end line which is y_1 =13cm, similarly y_2 = 7cm and y_3 = 1cm, so substituting the values we get

$$\bar{y} = \frac{12*13+20*7+20*1}{12+20+20} = 6.08\text{cm}$$

Now we have found out the $\bar{x}$ and $\bar{y}$ value that is $\bar{x}$ and $\bar{y}$ coordinates of the CG of the given I section.

Now let us calculate the moment of inertia of the given I section about its centroidal XX and YY axis

Now let us calculate the moment of inertia of the whole I section about its centroidal XX axis, here we have 3 axis we apply parallel axis theorem

$$I_{XX} = I_{G1} + a_2h_1^2 + I_{G2} + a_2h_2^2 + I_{G3} + a_3h_3^2$$

$$I_{xx} = \frac{6*2^3}{12} + 12*(6.92)^2 + \frac{2*10^3}{12} + 20*(7-6.08)^2 + \frac{10*2^3}{12} + 20*(6.08-1)^2$$

$$I_{xx} = 1285.03\text{cm}^4$$

Likewise if you do the calculation the moment of inertia of the whole I section about its centroidal YY axis, you will be getting the result as

$I_{yy} = 209.33 cm^4$.