B. ARCHITECTURE

MECHANICS OF STRUCTURES – 1 (AR6201)

PROPERTIES OF SECTION

Lecture - 7

Centre of Gravity and Centroid:

Now let us see the meaning of centre of gravity, n centre of gravity is a point through which the entire weight of an object acts irrespective of its position, generally centre of gravity is written as c.g, a body has only one c.g. Now consider this object in the image shown below

So this particular object will have its centre of gravity through the centre of gravity the entire weight of this object acts in the downward direction. WZ is the weight of the object, through the centre of gravity the entire weight of this body or object acts in the downward direction. Now if you change orientation of the object say like the image shown below. Even the orientation of the object change the centre of gravity will not shift or it will not be at some other. Centre of gravity will be remaining same, so always through the centre of gravity of an object its entire weight will be acting in the downward direction, so it is the point through which the entire weight of the object acts in the download direction irrespective of its position, also a body will have only one centre of gravity.

Centroid

Centroid is slightly different from centre of gravity; plane geometrical figures like rectangle, triangle, trapezium have areas but no masses. Centre of area of such figures is also known as centroid. Now the difference between centre of gravity and centroid is that centre of gravity indicates the centre of mass of solid objects or three dimension objects whereas centroid is the centre of area of two dimensional plane figures like square, trapezium, circle etc.

So consider the above image to be a circular plane section of certain diameter. Now area of the circle section will be

$$\text{Area} = \frac{\pi}{4} d^2$$

Centre of area of this plane section will be its centroid, so here the point G denotes the centroid of this circular section but in general both centre of gravity and the term centroid will be used in conjunction, there is nothing wrong if you say if use the term centre of gravity even for plain geometrical figures.

Axis of reference

Now let us see about axis of reference, generally the position of centre of gravity or centroid of plane figures are calculated with reference to some assumed axis. For plane figures the reference axis are taken as the lowest line and the left line of the figure.

Now consider a square section. Centre of area of this square section will be centroid or centre of gravity, so CG is the centre of gravity. Now generally the position or coordinates of the centre of gravity will be given with respect to some reference axis. The reference axis will be the lowest line of the figure, so this is the lowest line of the figure or we can call it as x axis and the left line of the figure, this is the particular line, left line of the figure can also be indicated as y. So these two are reference axis, that is x axis which is the lowermost line and y axis which is the leftmost line of the figure. So the position of the centre of gravity of this object will be found out with respect to these two reference axis.

Centre of gravity of plane figures:

Now let us see how to locate centre of gravity of plane geometrical figures, first of all we shall consider a square section, a square is a2 dimensional plane figure let the side dimension of a square section be 'a', for a square section centre of gravity is a point where the two diagonals of the square intersect one another so this is one diagonal this is the other diagonal, so CG is a point where the two diagonals are meeting, so this will be the CG.

For a rectangular section also the centre of gravity will be the point where the two diagonals of the rectangle are going to intersect. Now we shall to come triangular section. Let the base width of the triangle be b and height of the triangular section be h. For a triangular section centre of gravity will be the point where the three medians of the rectangle are going to intersect, median is the line which is drawn from the apex of the triangular to the midpoint of the opposite side. Let us name the corners of the triangle as A, B and C, so the first median will be from C to the midpoint of A, B. So this is that median another median will be from the corner B to the midpoint of A, C and the third one will be from A to the midpoint of BC, so these are the three medians centre of gravity will be the point where are the three medians are going to intersect with one another. Now centre of gravity for this triangular section from the base will be at a distance of h/3, remember h is the altitude or height of the triangle and the distance of CG of this triangle from its base will be h/3. Similarly for the previous square section distance of the CG from the base it should be naturally be a/2, similarly distance of the CG from the left line will also be a/2 this is for a square section

Now let us locate the centre of gravity of a semicircle, Now this semicircle as a diameter of d, now this dot represents the midpoint of the base of the semicircle, CG of this semicircle will be located at a height of or at a distance of 4r/3π right above the midpoint of the base of the semicircle, so this is the CG distance of the CG from the base will be equal to 4r/3 π and this CG will be right above the midpoint of the base.

Now let us consider a trapezium, a trapezium will have two sides parallel to one another. Now let b be the dimension of one of the parallel sides of the trapezium then the dimension of the other side other parallel side of the trapezium be ‘a’. Let the distance between the two parallel sides of the trapezium be ‘h’, this will be the CG of the trapezium, distance of CG from the base will be equal to

$$= \frac{h}{3}\left[\frac{b+2a}{b+a}\right]$$

So this is a distance of CG of the trapezium from the base, base being the longer side.

Centre of gravity of plane figures by method of moments:

Now Let us see how to calculate or how to locate the centre of gravity of plane geometricalfigures by method of moments. Consider this lamina, nowour aim is to locate the centre of gravity of this laminar figure with respect to this x and y axis that is we need to find out the coordinates of CG of this lamina area with respect to x and y axis for which we are going to adopt this method, this method is known as method of moments. What we have to do we have to split this lamina area into numerous small strips of much smaller area, to smaller subdivided areas along their own centre of gravity.

Now let the distance of centre of gravity of, so this will be the first area and this will be the second subdivided area. Let the distance of centre of gravity of the first subdivided area from Y axis be x_1 likewise let the distance between the CG of 2nd area small area from y axis be x_2. Like this we will be having n number of small strip areas, centre of gravity distance of centre of gravity of each and every areas from y axis will be x_1, x_2, x_3 upto x_n. The distance of centre of gravity of first area from x axis be y_1 and the distance of centre of gravity of the next area that is second area from x axis be y_2. Likewise we will be having distances up to y_n. So we have divided the given lamina area into numerous small steps and we have found out the coordinates of the centre of gravity of each and every divided small strips (x_1, x_2) will be the coordinates of centre of gravity of the first strip, similarly(x_2, y_2) will be the coordinates of centre of gravity of the second strip, likewise we will be have n number of small strips, the nth strip will have coordinates of (x_n, y_n). Let the x coordinate of CG of whole figure be $\bar{y}$ and the y coordinate of the CG of whole figure from x axis be $\bar{y}$, so coordinate of the CG be ($\bar{x}$, $\bar{y}$), so these two coordinate $\bar{x}$ and $\bar{y}$ have to be determined.

$$\bar{x} = \frac{a_1x_1 + a_2x_2 + ... + a_na_n}{a_1 + a_2 + ... + a_n}$$

Where a_1, a_2,…. areas of small strip, likewise we can also get y coordinate area.

$$\bar{y} = \frac{a_1y_1 + a_2y_2 + ... + a_ny_n}{a_1 + a_2 + ... + a_n}$$

Now this method can be adopted to locate the centre of gravity areas of other than conventional geometrical plane areas like square, rectangle, trapezium etc.

Problems for Centre of Gravity:

Now we shall see a problem for locating the centre of gravity of an unequal angle section, now these type of angle section will be used for the members of trusses and also as column sections. Now the dimensions of the unequal angle sections are given in the figure 1 leg of the angle section is 8 cm wide and the other leg is 10 cm wide, thickness of both the legs are 3 cm. We are asked to determine the position of centre of gravity of the unequal angle section for which we are going to adopt method of moments

Let me draw the same angle section here, if you see this unequal angle section it is a combination of two rectangles, so we shall splits this angle section into two rectangles by drawing a line like this, now we have got two rectangular section, let us number the rectangular section this will be rectangle number 1 and this will be rectangle number 2. As we discussed before CG will be located with respect to the reference axis. The reference axis will be the bottom most line of the figure which will be x axis and the left line of the figure which will be the YY axis. Now rectangle one will have its own CG the dot which I'm asking here represents CG of rectangle number 1. Now we shall consider the first rectangle. Area of the first rectangle is

$$a_1 = 8 * 3 = 24 \mathrm{cm}^2$$

The y coordinate of CG of rectangle one from x axis will be y_1 so this distance will be y_1 so the distance will be

$$y_1 = 7 + 1.5 = 8.5 \mathrm{cm}$$

Now we shall come to second rectangle which is rectangle number 2, area of the rectangle 2 will be

$$a_2 = 7 * 3 = 21 \mathrm{cm}^2$$

Now this rectangle 2 will have its own CG, this dot which I am marking here is the CG of rectangle 2 alone. Distance of CG to from x axis is y_2, y_2 is

$$y_2 = \frac{7}{2} = 3.5 \mathrm{cm}$$

Now we shall calculate the value of $\overline{y}$

$$\overline{y} = \frac{a_1 y_1 + a_2 a_2}{a_1 + a_2}$$

$$\overline{y} = \frac{(24 * 8.5) + (21 * 3.5)}{24 + 21} = 6.167 \mathrm{cm}$$

Now we shall calculate the value of $\overline{x}$, Now this is rectangle 1 , distance of centre gravity of rectangle 1 from y axis is will be denoted as $\overline{x}$, so X_1 is 8/4 = 4cm.

Likewise we shall get the value of distance of centre of gravity of rectangle 2 from y axis which will be X_2, X_2 will be

$$x_2 = \frac{3}{2} = 1.5\text{cm}$$

$$\overline{x} = \frac{a_1x_1 + a_2x_2}{a_1 + a_2}$$

$$\overline{x} = \frac{(24*4)+(21*1.5)}{24+21} = 2.833\text{cm}$$

So $\overline{x}$ for the figure is 2.833 cm and $\overline{y}$ is 6.167 cm.

Moment of inertia of Plane Area:

Consider a plane area like the above image and also consider a reference line. Now we are going to compute the moment inertia of this plane area with respect to this reference line which is AB, What we are going to do? once again we are going to divide the given area into numerous smaller areas smaller strips. Here I am showing only three strips entire area will be divide into strips. Now this dot represents the centre of gravity of the first strip, second dot is the centre of gravity of the second strip, third dot is the centre of gravity of the third strip. Now the distance of centre of gravity of the first strip from the reference line A, B is r_1, similarly let r_2 be the distance of the second strip from the reference line, Likewise r_3 be the distance of the third strip from the reference line, Likewise we will have distances upto r_n. Now moment of inertia of the given plane figure about the reference line AB will be given by

$$I_{AB} = a_1r_1^2 + a_2r_2^2 + \ldots\ldots + a_nr_n^2$$

Where a_1, a_2,…..a_n Areas of the small strip. From the equation for moment of inertia of this plane area we can say that moment of inertia is the second moment of area because moment of inertia is

$$I = \Sigma ar^2$$

Where is the area of the divided strips and r is the square of perpendicular distance between the CG of the divided strips from the reference line. Moment of inertia of a area will have the following units cm^4, mm^4, m^4.

Moment of inertia of areas by integration:

Consider a lamina figures, here reference axis are x axis and y axis in the previous case the reference axis was AB, now here the reference axis are x and y axis. Now we are going to consider a small strip area with in this lamina figure, this strip will have its centre of gravity, so this particular dot represents the centre of gravity of the strip area. Let the distance of the centre of gravity of the strip from y axis be x and the distance of the centre of gravity of the strip from x axis be y. moment of inertia of the strip about x axis or xx axis will be given by

$=dA*y^2$

Where dA is the area of the strip. We can calculate the moment of inertia of the whole lamina area about x axis which will be given by

$$I_{xx} = \Sigma dAy^2$$

Moment of inertia of the whole lamina area about y axis which will be given by

$$I_{yy} = \Sigma dAx^2$$