B. ARCHITECTURE

MECHANICS OF STRUCTURE – 1 (AR6201)

ANALYSIS OF PLANE TRUSSES

Lecture - 4

Introduction to Frames:

A frame may be defined as a structure made up of several members riveted or welded together. Truss is a frame in which the members may be of angle or channel sections. A simple form of truss is a triangle as shown in the fig. Now a truss is a frame which comprises of several members that are welded or riveted to each other. Now this is a very simplest form of truss in which there are 3 members. This is member 1, this is member 2 and the bottom member is member 3. Now the point where the two members are meeting is known as joints. Here for this triangular truss, there are 3 joints. At the ends you have support. So these two arrow marks indicate the supports for the truss. So this is a very basic and simple form of truss in which there are 3 members and 3 joints. The connection between the 3 members may be in the form of welded joint or riveted joint.

A statically determinate frame is a one in which the unknown member forces can be determined by using the static equilibrium equations. Basically there are 3 static equilibrium equations which are as follows. Algebraic sum of all vertical forces equal to 0, algebraic sum of all horizontal forces equal to 0 i.e. $\sum H = 0$ and the third equation would be algebraic sum of moments of all forces about a point in the system will be equal to 0 i.e. $\sum M = 0$. So $\sum V = 0$ Algebraic sum of all vertical forces equal to 0, and $\sum H = 0$ and $\sum M = 0$ are the three equilibrium equations. So in case of statically determinate frames, the unknown member forces can be determined by using the 3 conditions of equilibrium.

Now in case of truss, let us consider a simple form of truss which is a triangular truss. These are the two supports. Now p is the external load

acting on the truss. Generally it is assumed that load will be acting on the joints not within at intermediate point on the member. So the load p will be assumed to be acting only at the joint points because of this external load p. Each and every member of the truss will be subjected to some amount of stresses or load. So the unknown member forces in these 3 members shall be determined by using the three equations of equilibrium namely $\sum V = 0$, $\sum H = 0$ and $\sum M = 0$.

So if you are able to determine the unknown member forces by using these 3 conditions of equilibrium then such frames or such trusses are called as statically determinate frames or statically determinate structures. Now these statically determinate frames are composed of members which are just sufficient enough to keep the members in equilibrium without undergoing any distortion or excessive deformation. Also the statically determinate frames will satisfy the following condition. Number of members in the frame m will be equal to 2j-3 where in this equation, m is the number of members in the frame or truss and j is the number of joints. Now let us consider this particular simple form of truss which is triangular truss. Now there are 3 members 1,2 and 3. So number of members is equal t 3 which should be equal to 2 x number of joints. Number of joints are this is joint number 1, this is the second joint which is joint 2, and this is the final and 3rd joint number 3. So we have 3 joints so 2x3-3 which comes as 3. So number of members m satisfies this condition 2j-3 and hence this structure or this frame is known as statically determinate frame. Now let us see about statically indeterminate frames.

Now statically indeterminate frames are frames in which unknown member forces cannot be determined by using the conditions of static equilibrium alone. For determining the unknown member forces in addition to the 3 conditions of equilibrium equations in terms of deflection known as comparability equations are also needed for determining the unknown number of forces. Now for our study we are going to see the analysis of only statically determinate frames and trusses. Now let us see the assumptions made in the analysis of frames or trusses. All the joints in the frame are pin jointed. The other name of pin joint is hinged joint. Now these pin joint or hinged joint will allow rotation and hence there won't be any moment induced at the joints of the frame or truss. Frame is loaded only at its joints. This we have seen already so no load will be acting anywhere in the

intermediate part of the member so the load will be acting only at the joints. The frame is a statically determinate one. Weight of the members are assumed to be negligible when compared to the external load and hence they are neglected. Forces induced in the members are axial so because of the external load there will be some induced forces in the members. That member forces are truly axial in nature. Our aim is to determine the unknown member forces in the trusses because of this external load. All the members are in single plane which means that the truss is a 2Dimensional plane truss.

Types of Support for Trusses:

Now let us see the types of support for trusses. The first support is fixed support. the fixed support will be represented like this. In case of fixed support, there will be totally 3 unknown reaction components, there will be a vertical unknown reaction component V, there will be one horizontal unknown reaction component H, and also there will be a moment induced. This moment is M. So basically there are 3 unknown reaction components in case of fixed support. The next is simply supported ends for the truss. Simply supported ends is represented by a rectangle resting on its ends like this. In case of simply supported end, totally there will be 2 unknown reaction components which are vertical reaction component V and one horizontal reaction component H. Simply supported ends allows rotation and hence there won't be any moment.

So in case of simply supported end, moment will be 0 because it allows rotation. The next type of support is hinged support. The hinged support is represented like this, a triangle resting on its base, the apex of the triangle will be putting a dot. This dot represents the hinged end or hinged support. Once again the hinged support allows rotation and hence moment will be 0 and the two reaction components are the vertical reaction component V and the horizontal reaction component H. The final support is roller support, the roller support will be indicated by a triangle resting on a roller like this. Now for such kind of support there will be only one reaction component which will be vertical reaction component V.

Now this roller support allows rotation and hence moment will be 0. Also the roller support allows horizontal displacement so this support can be displaced towards right or it can be displaced towards left depending upon the direction from which the external load is applied. And hence since it is

allowing horizontal displacement there won't be any horizontal reaction and hence H will be 0. So the only unknown reaction component is V so the total unknown reaction component for roller support is 1.

Now we shall see the analysis of truss by the method of joints. Now the following points should be considered while doing the analysis of truss by doing the method of joints. Now we will be considering each joint in a truss as a separate free body in equilibrium and for that considered joint, we will be applying two equation of equilibrium namely $\sum V = 0$ and $\sum H = 0$. $\sum V = 0$ means sum of all vertical forces at the joint will be 0 and $\sum H = 0$ means sum of all horizontal forces at the joint will be 0. So by using these two equations of equilibrium the unknown forces at the joint under consideration will be determined. Now the joint that are considered as free body should be such that it should not have more than two unknown forces. This is the important point that has to be noted while doing the analysis by the method of joints.

Problem for Trusses:

Now let us solve a problem so that we can understand the method joints in a more elaborate way.

Now this is a simple truss, a basic triangular truss, span of the truss is 5m. It comprises of 3 members and 3 joints. the joints are named as A, B and C. At the apex of the truss i.e. at joint C we have a vertical downward force of 2t. Because of this force, each and every member of this truss, the members are AB, BC and AC will be subjected to some amount of forces. Those forces may be compressive force or tensile force. Now let us consider this as a member. As we discussed before Members will be made up of angle sections or channel sections. Consider this as member. Now this member is subjected to axial compressive force of magnitude p then within the member there will be an equivalent opposite reaction offered for the stability of the member. So equal and opposite reaction will be offered in the direction opposite to the applied compressive force.

So the broken arrow indicated the reaction forces. Now for the analysis of truss, we will be indicating the member by a single line like this and at the ends of the member we have joints, the joints will be denoted by two dots. So a member has two joints at its ends. Now in case if this member is subjected to compressive force then the resisting force the arrow mark

should be such that it should be pointing like that of the resisting force. So compressive force acting on the member will be indicated like this. Arrow mark should be pointing towards the joint i.e. in case if the member is subjected to compressive force the joints will be pushed away from each other.

Now if a member is subjected to a tensile force of magnitude p so this is the member which is subjected to a tensile force of magnitude p, an equal and opposite reaction will be offered within the member. So the reaction force will be exactly opposite to the applied tensile force. Now this reaction force is offered for the stability of the member. So a member subjected to tensile force will be indicated like this. So the member is indicated by a line. At the ends we have two joints. The arrow mark will be pointing away from the joints like this.

So this is how we denote tensile force i.e. the two joints will be pulled by the tensile force towards one another. Now let us come to the problem. The truss has simply supported ends. Now first of all we have to determine the unknown reactions. the unknown support reactions. Now at the end A we have the unknown reaction component V_A and at the end B we have the unknown vertical reaction component V_B. now at the two ends there won't be any horizontal reaction component because if you see the nature of external load the 2ton external load is pure vertical external load and hence at the two simply supported ends there will be only vertical reaction components. Now we are going to determine the two vertical reaction components for which we are going to use the conditions of equilibrium.

$$\sum V = 0$$

for the equilibrium of the structure for the truss, algebraic sum of all vertical forces will be equal to 0. We are going to take all the vertical forces in the upward direction as positive and hence forces going in downward direction as –ve. Now totally there are 3 forces in the truss. The three forces are V_A V_B and external force 2ton. V_A goes in upward direction. So we have to put a +ve sign for it similarly V_B is in the upward direction and 2ton force the external force is in downward direction so $+V_A+V_B-2=0$. So $V_A+V_B=2$. Let us keep this equation as number 1.

Now algebraic sum of moments of all forces about any point. Say let us consider this joint A, this joint A sum of moments of all forces should be

equal to 0 because joint A is a pin joint and hence there won't be any moment. $\sum M_A = 0.$ Similarly $\sum M_B = 0.$ So you can also use the second equation as $\sum M_B = 0.$

Now once again 3 forces are there V_A, V_B and 2ton. V_A passes through A and hence there is no leave arm for this force. There won't be any moment due to V_A about A. Now let us consider the force V_B. Now here is our joint A, V_B acts like this, V_B is in the upward direction. So V_B induces the anticlockwise moment about A.

We are going to use the following sign conventions for moments. Anticlockwise moments will be –ve. So we will be using –ve sign for all anticlockwise moments. So V_B induces anticlockwise moments so $-V_B x5$. Now the third force is 2ton now if we see the inclination of the two sloping members they are at 45^0 inclinations so the perpendicular distance between this 2ton force and joint A is 2.5m because this triangular truss is in the form of an isosceles triangle. So this distance will be 2.5m. Now 2ton force creates a clockwise moment A. Clockwise moment will be +ve so it will be $+2x2.5=0$. $V_B=5/5 = 1$ton. Now already we have seen that substituting the value of V_B in 1we get $V_A+1=2$. Therefore $V_A=1$ton. So the two unknown reaction components are of equal magnitudes 1 ton each. Now let us consider a free body of joint A. Now this is joint A, at joint A you have vertical reaction component V_A. Magnitude of V_A is 1ton then you have the horizontal member, horizontal member is AB and you have an inclined member which is AC. Let us indicate the force in the member AB as F_{AB} and force in the member AC as F_{AC}.

Now our aim is to determine the force in the member F_{AB} and force in the member F_{AC}. The inclination between these member which is also given as 45^0. Now these 2 members AC and AB maybe subjected to any nature of stress i.e. it can be subjected to either tensile force or it can be subjected to compressive force. Generally all the bottom chord members of the truss will be subjected to tensile force. As we have seen earlier tensile force will be indicated by an arrow mark pointing away from the joint. So F_{AB} is assumed to be tensile in nature and hence i'm putting an arrow mark pointing away from the joint. Now we shall assume force in the member AC to be compressive in nature. Compressive forces will be indicated by drawing an arrow mark pointing towards the joint. I've shown an arrow mark pointing towards the joint so force in AB is assumed to be tensile and force is AC is

assumed to be compressive. Now this is joint A. Now we are going to apply the two conditions of equilibrium namely $\sum H = 0$ and $\sum V = 0$ for determining the unknown forces. First of all we shall apply the condition $\sum V = 0$. Now at joint A, we have a vertical force V_A, V_A acts in the upward direction and hence the sign convention for that force V_A will be positive so $+V_A$. Now we have to find out the vertical component of the force F_{AC}. This is the force F_{CA} it acts towards the joint.

Let us construct a triangle, right angled triangle with F_{CA} as hypotenuse. This angle is 90^0 and the inclination of F_{CA} with horizontal is 45^0. This is the vertical component of F_{CA} and this is the horizontal component. So F_{CA}, force in CA acts in this particular point and it ends up at the joint A i.e. it moves from this particular point to this particular point. In order to travel in this manner vertically we have to travel downwards and horizontally we have to move towards left so the direction of vertical component of F_{AC} is in the downward direction and the direction of horizontal component is towards left. Now the vertical component will be in –ve sign i.e. the vertical component will be in downward direction so it will be $-F_{CA}\sin 45$. So $+V_A - F_{CA}\sin 45 = 0$. Now we know the value of V_A so therefore F_{CA} will be equal to $1/\sin 45$ so you get F_{CA} as 1.414ton. Now if you see the sign for F_{CA} we are getting a positive sign which means that the assumed nature of forces for the member AC i.e. compressive is correct so the assumption that the force of the member AC is right because of we get the sign as positive. So F_{AC} is subjected to a compressive force of magnitude 1.414 ton. Now we shall apply the second condition of equilibrium i.e. $\sum H = 0$ for joint A.

Now if you consider this joint A, totally there are 3 forces coming at joint A. V_A is vertical force and hence it does not have any horizontal component. Now let us consider the force F_{AB}. F_{AB} goes towards right so the horizontal component of F_{AB} will be $+ F_{AB}\cos 0$ because F_{AB} is purely horizontal in nature. So $F_{AB}\cos 0$ will be once again $F_{AB.}$ So $+F_{AB}$. Now we have to get the horizontal components of the force $F_{AC.}$ Now the horizontal components of the force F_{AC} is here below this triangle which is represented as H. That horizontal component if you see the direction of horizontal component which is towards left. It is towards left so we have to put a negative sign. Horizontal component of F_{AC} will be $F_{CA}x\cos 45$.

So these are the algebraic sum of all horizontal forces. Equate this to 0. Already we have found out the value of F_{CA} which is 1.414 ton and upon substitution and simplifying we will be getting F_{AB} as 1ton. Once again if you see the sign convention for F_{AB} you get a positive sign which means that our assumed nature of force for the member AB i.e. the tensile is right and F_{AB} is subjected to a tensile force of 1ton. By considering the equilibrium of joint A we have found out 2 unknown member forces namely F_{AB} and F_{CA}. Similarly we can consider the equilibrium of joint C and then we can determine the remaining unknown member force which is F_{CB}. Now if you do the calculation, F_{CA} will be exactly equal to F_{CB} because of symmetrical loading.