Mechanics of Structure – I

Lecture 3

Principle of moments: When an object which is acted upon by several forces is in equilibrium, the sum of all clockwise moments will be equal to the sum of all anticlockwise moments. Let us consider 2-Dimensional plane figure which is subjected to several forces like this. Now this plane of figure is subjected to 3 forces namely F_1, F_2 and F_3. Perpendicular distance of these 3 forces with respect to a point O. Now this is the point O. Perpendicular distance of these 3 forces with respect to point O are r_1, r_2 and r_3 i.e. perpendicular distance between O and F_1 is r_1 and perpendicular distance between O and F_2 is r_2 and perpendicular distance between O and F_3 is r_3. Now according to the principle of equilibrium, here this object is subjected to 3 forces and if this object is in equilibrium then algebraic sum of all clockwise moments about O will be equal to algebraic sum of all anticlockwise moments. Hence according to this theorem, now let us consider the force F_1. Force F_1 about O creates a clockwise moment. Now this is point O and the force F_1 acts like this, so it induces a clockwise moment with respect to O. So let us keep clockwise moments as +ve. Now let us compute all the clockwise moments. So we are computing all the clockwise moments first. F_1 creates a clockwise moments about O, perpendicular distance between F_1 and point O is r_1 so the clockwise moment would be F_1xr_1. Now let us consider the second force i.e. F_2. Here is the point O, F_2 acts like this once again F_2 creates a clockwise moment with respect to point O. So to the first clockwise moment F_1xr_1 we will be adding the moment due to F_2. So it is F_2xr_2. So F_1 and F_2 are the two forces which are giving clockwise moments with respect to O. Now let us consider the force F_3, so here is our point O, F_3 acts like this, so it induces an anticlockwise moment with respect to O. F_3 is the only force which gives the anticlockwise moment. That anticlockwise moment with respect to O will be F_3xr_3. Since this plane of figure is in equilibrium, sum of all clockwise moments will be equal to sum of all anticlockwise moments and hence $F_1r_1+F_2r_2=F_3r_3$. This is known as principle of moments.

Now we shall see about Varignon's theorem. According to this Varignon's theorem moment of the resultant of all the forces about a point is equal to the algebraic sum of moments of all the forces about that point. Now let us consider 2 forces meeting at a point, the resultant of these 2 forces will be like this. The resultant of these 2 forces F_1 and F_2 will be the diagonal of the parallelogram passing through the intersection point O. So R is the resultant here. Now let us consider any point P. Perpendicular distance between the force F_1 and P be r_1, Perpendicular distance between the resultant R and P be r and Perpendicular distance between the force F_2 and P be r_2. So this angle will also be 90^0. Similarly this angle is 90^0. According to Varignon's theorem moment of the resultant of all forces P, here the resultant is R and the moment of the resultant P will be Rxr. Moment of resultant of all the forces about P will be equal to algebraic sum of moments of all forces about P. algebraic sum of all forces about P will be $F_1xr_1+F_2xr_2$. Here for both these moments +ve sign is coming because both F_1 and F_2 will induce clockwise moments with respect to point P and hence we have +ve sign for those two moments. Similarly the resultant R will also induce clockwise moment with respect to P and hence +ve sign is coming for this resultant moment. So this is the statement according to Varignon's theorem.

Now we shall see the principle of moments and varignon's theorem to solve some problems. Let us see the 1^{st} problem. Four parallel forces of magnitudes 100N, 200N, 50N and 400N acts on a straight line as shown in the figure. Determine the magnitude of the resultant and also the distance of the resultant from point A. So in this figure there is a line ABCD, over the line ABCD we have 4 parallel forces acting, these are unlike parallel forces magnitude of the four forces are shown in the figure also the distances between the four forces are also shown in this figure. We are asked to determine the magnitude of the resultant and also its position. Magnitude of the resultant will be easily obtained by determining the algebraic sum of all vertical forces. Algebraic sum of all forces. But in order to determine the position of the resultant we have to use the varignon's theorem. Now let's see the solution for this. So this is the line ABCD in which 4 forces are acting. Distance between the force are given and the resultant of all the forces will be computed like this. We will be taking all the forces going in the upward direction as +ve and hence forces going in the downward direction is taken as –ve sign. Now we shall consider the 1st force which is 100N which is going in the upward direction so +ve sign will be coming for that 100N, the next force 200N goes in the downward direction so –ve sign will be coming for that force similarly –ve sign will be coming for the 3rd force which is 50N and finally the 4th one goes in the upward direction, 400N goes in the upward direction so +ve sign will be coming for that force. So basically the resultant for parallel forces will be the algebraic sum of forces, so here the four forces are parallel forces and hence the resultant is nothing but algebraic sum of all the forces. So the resultant works out to be 250N. If you see the sign of the resultant it comes as +ve sign and hence the direction of the resultant will be in the upward direction. So let us mark the resultant on the line ABCD, the resultant may be acting anywhere between AB or BC or it can be between C and D. We don't know the exact position of that resultant so what we are going to do, I'm going to arbitrarily assume the position of the resultant. It is somewhere between B and C. So this is the resultant. Magnitude of the resultant is 250N. Resultant acts in the upward direction let the distance between the resultant and point A be x. I'm denoting the distance between the resultant and point A as x. So by varignon's theorem moment of the resultant about a certain point here I'm going to consider point A, moment of the resultant about point A will be equal to the algebraic sum of the moments of all other forces about point A. Moment of R about point A will be 250, perpendicular distance between R and point A is x. So 250x. Now this is point A, resultant acts like this, so about point A, R induces anticlockwise moment so I'm going to put a –ve sign for this anticlockwise moment. So this is the moment of resultant about A this should be equated to algebraic sum of all other forces about A. Now let us consider this 100N force 1st. 100N force passes through A and hence it does not have any lever arm i.e. perpendicular distance hence moment due to 100N force about A will be 0. Next we shall consider this 200N force, 200N force creates, this is point A 200N force acting like this, it induces a clockwise moment, anticlockwise moment is taken as –ve and hence clockwise moment will be taken as +ve so it will be 200 into the perpendicular distance between 200N and A is 1m so into 1. Similarly for 50N force, 50N force creates a clockwise moment about A, so the moment will be 50x perpendicular distance will be 1m+1.5m which is 2.5m and finally this 400N force creates an anticlockwise moment about A so anticlockwise moment we have to put –ve sign so 400x perpendicular distance between 400N and A is 3.5m. So from this if you calculate x comes as 4.3m from the point A. So this x is calculated as 4.3m. For this given force system we have calculated the magnitude of the resultant as well as its position with respect to the point A. For this problem we have applied the varignon's theorem.

We shall see one more problem. Four forces of magnitudes 20N, 40N, 60N and 80N are acting respectively along the four sides of a square ABCD as shown in the figure. Determine the resultant moment about the point A. Each side of the square is 2m. The figure shows a square ABCD of side 2m, along the four sides we have 4 different forces acting, along the side AB we have the magnitude of force 20N, along the side BC we have the magnitude of force 40N, along the side CD we have the magnitude of force 60N, and finally along DA we have 80N force. We are asked to determine the resultant moment of all the forces about point A. The resultant moment of all the forces will be the algebraic sum of moments of all forces about point A. We shall construct the square along with the four forces. Resultant moment about point A will be equal to algebraic sum of moments of all forces about A. Once again we are going to use varignon's theorem. First of all we shall consider this 80N force, 80N force passes through point A and hence it does not have any perpendicular distance i.e. lever arm so the moment due to 80N force about A will be 0. Next we shall consider this 20N force, if you see this

20N force this 20N force is also passing through point A and hence it does not have lever arm so the moment due to 20N force about A will also be 0. Now let us consider this 40N force, 40N force we shall extend that force so this is the line of action of this 40N force, perpendicular distance between 40N force and point A is this distance which is the side dimension of the square which is 2m so naturally 40N force creates a moment about A, that moment will be an anticlockwise moment, so the moment will be 40x2. Since it is an anticlockwise moment we shall put a –ve sign here. Now let us consider the final force which is 60N force. 60N force acts along the side DC, it does not pass through point A. perpendicular distance between 60N force and point A is once again the side dimension of the square which is 2m, 60N force once again creates an anticlockwise moment about A and hence we shall put a –ve sign so -60x2 so we get -200Nm. This is the resultant moment of all the four forces about point A. –ve sign indicated that the nature of resultant moment is anticlockwise.