Mechanics of Structure – I

Lecture 2

First of all we shall see law of parallelogram of forces. The statement for this law is as follows. When two forces meeting at a point are represented in magnitude and direction by the two adjacent sides of a parallelogram then the resultant of the two forces is represented in magnitude and direction by the diagonal of the parallelogram passing through that point.

Now let us consider two concurrent forces namely P and Q. Now these two forces are intersecting at the point O. Now according to the law of parallelogram of forces if we are able to represent these two forces by the two adjacent sides of a parallelogram. Now in this figure you can see that the forces P and Q forms the two adjacent sides of a parallelogram. If we are able to represent the two forces by the two adjacent sides of the parallelogram then the resultant of these two forces are represented both in magnitude and direction by the diagonal of the parallelogram passing through the intersection point O. Now let us construct the diagonal of the parallelogram which passes through the point O. So this line will be the diagonal of the parallelogram which passes through the intersection point of P and Q. So length of the diagonal will give you the magnitude of the resultant of P and Q and the direction of R with respect to P is indicated as θ. So by this way we can calculate the resultant of two concurrent forces meeting at a point by using the law of parallelogram of forces. Now we shall name the four corners of the parallelogram. The intersection point of P and Q is O, let this point be A, the force Q is represented by QB and let this point be C. From C we shall drop a perpendicular. We shall extend the P to meet the perpendicular line. So this angle will be 90^0 and let us mark this point as D. The inclination between the two forces P and Q shall be represented as α. So α is the inclination between the forces OA and OB. Angle BOA and angle CAD will be equal. So this angle CAD will also be α. Now we shall get the magnitude of the resultant R.

Now let us consider the triangle OCD. ΔOCD is right angled triangle in which $OC^2=OD^2+CD^2$. So OC is our resultant R which we need to find out. OD is the sum of OA and AD. So $R^2= (OA+AD)^2+CD^2$. Now OA is the force P and we need to find out the distance AD. Now in the ΔCAD. Once again CAD is right angled triangle with angle CDA be 90^0. Now here the side AC is once again our force Q. So if you take $\cos\alpha=AD/AC$. Now we need AD. Hence $AD=AC\cos\alpha$ but AC is our force Q. So $AD=Q\cos\alpha$. So in this equation OA is P + we have to substitute the value of AD and AD has been computed as $Q\cos\alpha$. So $(P+Q\cos\alpha)^2+CD^2$. Now once again the right angled triangle CAD, if we take $\sin\alpha=CD/AC$. So CD is the required distance. So CD will be equal to $AC\sin\alpha$. AC is Q so it will be $Q\sin\alpha$. So we shall substitute the value of CD here. CD will be equal to $(Q\sin\alpha)^2$. So if we expand this equation, we will be getting

$R^2=P^2+Q^2\cos^2\alpha+2PQ\cos\alpha+Q^2\sin^2\alpha$. Now from these two terms we can take Q^2 as common so we will be getting $P^2+Q^2(\cos^2\alpha+ \sin^2\alpha)+ 2PQ\cos\alpha$. So $\cos^2\alpha+ \sin^2\alpha$ is 1. Therefore you get $P^2+Q^2+2PQ\cos\alpha$. or we shall get the value of resultant,

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}$$

So by applying the law of parallelogram of forces, we have found out the resultant. This resultant is nothing but diagonal of the parallelogram passing through the intersection point of the two forces P and Q.

Now we shall get the direction of the resultant which is represented as θ.

Direction of the resultant R, Now we shall consider this triangle ΔOCD. In this triangle if we take

$$\tan\theta = \frac{CD}{OD} = \frac{CD}{OA + AD}$$

$$= \frac{Q\sin\alpha}{P + Q\cos\alpha}$$

$$\theta = \tan^{-1}\left(\frac{Q\sin\alpha}{P + Q\cos\alpha}\right)$$

This is the direction of the resultant and the magnitude of the resultant is given here. By this way we can determine the direction as well as the magnitude of the resultant by applying law of parallelogram of forces for two concurrent forces meeting at a point.

Now we shall see the principle of equilibrium. A stationary object subjected to coplanar forces will be in equilibrium if the algebraic sum of all external forces is zero and the algebraic sum of moments of all forces about a point in that plane is zero. This is the statement for the principle of equilibrium.

Now let us consider this case. Now here this particular point is acted upon by several forces. Here we have 4 number of forces acting on a particular point. According to the principle of equilibrium. Now the point which is subjected to four forces or the body which is subjected to four forces will be in equilibrium if the algebraic sum of all the forces acting on this particular point or acting on this particular object is equal to zero which means that algebraic sum of all forces should be zero. Here algebraic sum of all the horizontal components of forces should be equal to zero and algebraic sum of all the vertical components of forces should be equal to zero. Already we have seen how to compute the vertical and horizontal components of given forces. So algebraic sum of all horizontal components and algebraic sum of all vertical components of all the forces should be zero. Moreover algebraic sum of moments of all forces about a particular point in that system should also be 0. So $\sum M$ is the algebraic sum of moments.

Now we shall learn about Lami's Theorem. If three forces meeting at a point in equilibrium, then each force will be proportional to the sine of the angle between the other two forces. Now here we have three concurrent forces F_1, F_2 and F_3 meeting at a point. Now if this force system is in equilibrium then according to Lami's theorem we can say that each force will be proportional to the sine of the angle between the other two forces. So

$$\frac{F_1}{\sin\beta} = \frac{F_2}{\sin\gamma} = \frac{F_3}{\sin\alpha}$$

Now if we consider the force F_1, angle between the other two forces is β. Similarly if we consider the force F_2, the angle between the other two forces is γ. If we consider the force F_3, the angle between the other two forces is α. So lami's theorem says that each force will be proportional to the sine of the angle between the other two forces. Accordingly we have

$$\frac{F_1}{\sin\beta} = \frac{F_2}{\sin\gamma} = \frac{F_3}{\sin\alpha}$$

This Lami's theorem can be used to determine unknown forces provided the number of concurrent forces are 3 and if the 3 concurrent forces are in equilibrium. By this way we can calculate the unknown forces in case of 3 concurrent forces.

Now we shall see how to calculate the moment of a force. We all know that force is a vector quantity. So it has certain magnitude as well as it has direction. Let F be the force. Now we can calculate the moment of this particular force about a certain point O. Moment of a force F about a particular point under consideration O is the, here moment is represented by M, so moment of force F about point O is the product of magnitude of the force and the perpendicular distance between the line of action of the force and the point O. Here r is the perpendicular distance between the line of action of the force and the point O, so the moment M is the product of force F and the perpendicular distance r. So r is the perpendicular distance between the force F and the point O.

Now let us solve a problem. Three forces of magnitudes P, 100N and 200N are acting at a point O as shown in the figure. Determine the magnitude and direction of force P if the system is in equilibrium. Here in this picture you can see three forces i.e. P is the unknown force which we have to find out, the direction of P with respect to positive X axis is θ so we are also required to find out the direction i.e. the angle θ. The other 2 forces are 100N force and 200N force. Inclination of 100N force with respect to –ve X axis is 30^0 and inclination of this 200N force with respect to –ve Y axis is 15^0. In this problem it is given that the force system is in equilibrium. So if the force system is in equilibrium, it should satisfy three condition i.e. algebraic sum of all horizontal components of the forces should be equal to 0 and algebraic sum of all vertical components of the forces should be equal to 0 and algebraic sum of moments of all forces about any point should be equal to 0. Now to get the value two unknowns P and Q, we will be using two conditions i.e. sum of all vertical components of the forces will be 0 and sum of all horizontal components of the forces will be equal to 0. Using those two conditions we can find out the value of P and θ. Here I've drawn the three forces and I've shown the inclination of forces with respect to different axis. Now since the system is in equilibrium algebraic sum of all horizontal components of the forces should be equal to 0 and algebraic sum of all vertical components of the forces should be equal to 0 i.e. $\sum F_H = 0$ and $\sum F_V = 0$. Now we shall get horizontal components of all the three forces. First of all we shall consider the force P. Now the force P starts from here and it ends up here, so horizontally we have to move towards right and then we will be reaching this particular point i.e. this point and vertically we will be moving in the upward direction. this is the vertical component of the force P and this is the horizontal component of the force P. So horizontal component is along X axis and vertical component is along Y axis. Inclination of P with respect to X axis is given as θ. Now the vertical component of P will be given by $P\sin\theta$. If we see the direction of this vertical component it is in upward direction. Generally we will be taking the forces going in upward direction as +ve. So the sign for the vertical component will be $+P\sin\theta$. Now we shall calculate the horizontal component of this force. If we see the direction of horizontal component, it is towards right. We will be taking all the forces towards right as +ve so $H=+P\cos\theta$. In the same manner we can get the horizontal and vertical components of this 100N force. So 100N force goes like this, here we have Y axis and this is the X axis. This is 100N force, inclination of 100N force with respect to X axis is 30^0. Now 100N force acts from the origin i.e. this point and it reaches here. So to move from this particular point to this particular point, horizontally we have to go towards left and we will be reaching this particular point and vertically we will be going in the upward direction. So this is the horizontal component. Direction of horizontal component is towards left and this is the vertical component. vertical component acts in the upward direction. So $H=-100\cos30$, vertical component V will be $V = +100\sin30$. Finally we have one more force to resolve which is 200N force. This 200N force is inclined at 15^0 with respect to Y axis. 200N force starts from the origin i.e. this particular point and it reaches here. So to move from this origin and to end up here vertically we have to go in the downward direction and we will be reaching this particular point and from here we have to move horizontally towards right. So here this is the horizontal component and this is the vertical component. So first of all we shall find out this horizontal component. Direction of horizontal component is towards right and hence we will be putting a +ve sign here. So with respect to 15^0 here the horizontal component H is the opposite side so the H will be $+200\sin15^0$, vertical component V will be, in the figure if you see the direction of vertical component, the vertical component goes in the direction of downward direction. All forces going in the downward direction will be negative so V will be $V=-200\cos15$. So we have found out the components of horizontal and vertical components of all the three given forces. Now we shall apply the conditions of equilibrium in order to find the unknown forces P and its inclination θ.

I'm applying the first condition of equilibrium which is summation of all horizontal components of forces and I'm equating it to 0. Already we have found out the horizontal components of the three forces. Now I've substituted the horizontal components of the three forces,

$+P\cos\theta-100\cos30^0+200\sin15^0=0$.

Now from this we are getting the value of $P\cos\theta$ as 34.84. Let us keep this as equation 1. Now we shall apply the next condition of equilibrium i.e. $\sum F_V = 0$

Here I've substituted the vertical components of three forces,

$+P\sin\theta+100\sin30^0-200\cos15^0=0$.

From this we are getting the value of $P\sin\theta=143.185$. We shall keep this as equation 2. Now we shall divide equation 2 by equation 1. we get $P\sin\theta/P\cos\theta$ which will give us $\tan\theta=4.11$. From this we can get the value of θ as $76^0 19'$. Substituting the value of θ in any one of the equations, equation 1 or 2, we will be getting the value of the unknown force P. Unknown force P comes to 147.36N.

So by applying the conditions of equilibrium we have found out the unknown force in a concurrent force system if the concurrent force system is in equilibrium