B. ARCHITECTURE

MECHANICS OF STRUCTURES – 1 (AR6201)

ELASTIC CONSTANTS

LECTURE - 13

Example Problem for Bulk Modulus:

A steel bar 25cm long and 5cm by 5cm in cross section is subjected to a tensile load of 3000KN along longitudinal axis and compressive loads of 500KN on each sides. Under the action of the above loads, the change in volume is absorbed to be $0.05cm^3$. Determine the value of Poisson's ratio and bulk modulus. Take young's modulus E = $2*10^5$ N/mm^2.

Now we have the bar the cross section of the bar is a square. The length of the bar is 25cm breadth is 5cm and thickness is also 5cm. now we consider the three axis so this will be x axis and this is y axis and this is z axis along the longitudinal axis we have tensile loads of 300KN. So tensile loads 300KN is acting along the longitudinal axis which will be the x axis. On the two other sides of the square bar we have compressive force of 500KN along z direction a compressive force of 500KN acting. Similarly along y axis compressive force of 500KN is acting. Changing the volume of the bar

δv = $0.05cm^3$.

You are has to determine the value of poissons ratio which is 1/m and the bulk modulus k these two are the quantities which are to be found out. Now we shall calculate the stress along x axis force along x axis is 300KN. It is tensile force we shall convert the force into newton $300*10^3$ divided by cross section area perpendicular to this 300KN tensile force 5*5. We shall substitute the value of cross section area in terms of cm^2.

Stress along x axis = $\frac{300*10^3}{50*50}$ = 120N/mm^2 (Tensile)

The nature of truss is tensile now we shall calculated the Stress along y axis. So this is direction of y axis along y axis we have compressive force of 500KN so stress along y axis will be force by area. Force is $500*10^3$divide by area perpendicular to the 500KN forces length of the bar 25cm cross the thickness.

Stress along y axis = $\frac{500*10^3}{250*50}$ = 40N/mm^2 (compressive)

Likewise we shall calculated the Stress along z axis will be this is z axis load acting z axis is 500KN compressive force $500*10^3$ area normal to that force is the 25cm cross 5cm.

Stress along z axis = $\frac{500*10^3}{250*50}$ = 40N/mm^2 (compressive)

So stress acting along x axis is denoted as P_x Stress acting along y axis is denoted P_y Stress acting along z axis is $P_{z.}$ After calculating the stress along three direction we shall calculated the value of strain along the respective axis Strain along x axis is given by

$$\varepsilon_x = \frac{P_x}{E}$$

Now once again going to previous slide if you see the nature of stress acting along three axis P_x is tensile that once again recollect the all tensile stress will have positive sign and compressive stress will have negative sign. So along y axis and z axis are compressive in nature and hence P_y and P_z alone will have negative sign. P_x is tensile stress and hence it will have positive sign.

Now for this equation that is Strain along x axis $\frac{P_x}{E}$. P_x is positive sign because of tensile stress P_y has compressive in nature so will be having and P_Z once again compressive in nature will be having another minus here

$$\varepsilon_x = \frac{P_x}{E} - \frac{(-P_y)}{mE} - \frac{(-P_z)}{mE}$$

Now we substitute the value of P_x, P_y, P_z and E here

$$\varepsilon_x = \frac{120}{E} + \frac{40}{mE} + \frac{40}{mE} = \frac{120}{E} + \frac{80}{mE}$$

So this is the value of ε_x likewise strain along y axis will be given by

$$\varepsilon_x = \frac{P_y}{E}$$

Where P_y is the direct stress acting along the y axis but that stress compressive in nature and we had put a minus here. Then minus of stress acting along two axis which will be strain acting along x axis and strain acting along z axis. Strain acting along x axis will be now since P_x is positive sign because of tensile stress. P_z have a compressive in nature so we put negative sign

$$\varepsilon_y = -\frac{P_y}{E} - \frac{P_x}{mE} - \frac{(-P_z)}{mE}$$

Once again substitute the value

$$\varepsilon_y = -\frac{40}{E} - \frac{120}{mE} + \frac{40}{mE} = -\frac{40}{E} - \frac{80}{mE}$$

Finally we get the value of strain along z axis

$$\varepsilon_Z = -\frac{P_Z}{E} - \frac{P_x}{mE} - \frac{(-P_y)}{mE} = -\frac{40}{E} - \frac{120}{mE} + \frac{40}{mE} = -\frac{40}{E} - \frac{80}{mE}$$

So ε_y and ε_z strain acting along y and z axis. Now the volumetric strain

$$\varepsilon_\nu = \frac{\delta\nu}{\nu} = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

$$\varepsilon_\nu = \frac{120}{E} + \frac{80}{mE} - \frac{40}{E} - \frac{80}{mE} - \frac{40}{E} - \frac{80}{mE} = \frac{40}{E} - \frac{80}{mE}$$

$$\varepsilon_v = \frac{40}{E}\left(1 - \frac{2}{m}\right)$$

So this is the volumetric strain

$$\varepsilon_v = \frac{\delta\upsilon}{\upsilon}$$

From this we shall get the value of

$$1 - \frac{2}{m} = \frac{\delta\upsilon}{\upsilon} * \frac{E}{40}$$

δv is given by 0.05 that is change in volume absorbed to be 0.05cm^3 volume should be substituted in cm^3

$$\frac{\delta\upsilon}{\upsilon} * \frac{E}{40} = \frac{0.05}{25*5*5} * \frac{2*10^5}{40}$$

$$1 - \frac{2}{m} = 0.4$$

$$\frac{1}{m} = 0.3$$

So this is the value of poissons ratio. Next we also have to calculate the value of bulk modulus of elasticity. So we shall use the relationship between young's and bulk modulus

$$E = 3K\left(1 - \frac{2}{m}\right)$$

We know the value of E is

$2*10^5$ = 3K (1-0.6)

Bulk modulus K = $1.66*10^5$ N/mm^2.

Definition of Shear Stress (q):

Definition for shear stress a section will be subjected to shear stress if the force is acting tangentially across it. Now let us as consider a cube which is fix at his base so A, B, C, D is the cube with the side AB rigidly fixed at his base. Over the top surface let us consider a force acting tangentially over it. That is at the top surface we have a force of magnitude P acting tangentially over it. Because of this tangential forces the side DC will be displaced to D′ and C′ like this whereas the side AB won't suffer any displacement because it is fixed at this bottom. So this cube will deform in this manner the angular deformation is denoted as θ, so let l be the side of the cube. So over the top surface DC we have tangential forces of magnitude P so the top surface is going to experience shear stress the intensity of shear stress as per the definition will be the magnitude of tangential force divide by the area of the phase DC or the top surface will be square one

$$Q = \frac{P}{l^2}$$

So the shear stress equal to tangential force divide by area resecting that tangential force. So P is tangential force or also call it as shear force. Now shear strain will be measured in term of the angular deformation so shear strain is θ this is the angular deformation which the top side which going to experience. Now tthe angular deformation is very small angle so if you take tan θ now this is the right-angle triangle in which you can take

$$\tan\theta = \frac{DD'}{AD}$$

The θ being very small approximate to

$$\theta = \frac{DD'}{AD} = \frac{CC'}{BC}$$

So this angular deformation is known as shear strain.

Definition of Shear Modulus or Modulus of Rigidity (N):

Definition for shear modulus or modulus of rigidity. Modulus of rigidity is denoted by N shear modulus is the ratio of shear stress q to shear strain θ so it will have a units will be N/mm^2 or kg/mm^2.

Relationship Between Young's Modulus (E) & Rigidity Modulus (N):

Now we the relationship between the young's modulus and rigidity modulus. Let us as consider a cube A, B, C, D the top surface is subjected to a shear stress of intensity q so q is the intensity of shear stress acting over the top surface because of the shear stress the side DC will be displaced the corner D will displaces D′ and the other corner C will displaces C′. So this angle is ∅ which is shear strain now let us join BD′ and BD by a full line. Now the corner D will displaces D′and the other corner C will displaces C′. The original diagonal is BD now this diagonal will be elongated to DD′ let us draw an arc of radius equal to the length of the diagonal BD from D so this is that arc. So radius of the arc is length is BD this is arc is drawn center point of this D. so let us name the particular point of D″. Now let us consider small triangle D′ D and D″. So this point will be D′ this point is D″ and here we have D the angle D′ D and D″ will be more or less 45^0.

Actually slightly less than 45^0 only by a very small amount hence this angle can be taken as 45^0. So this angle D′ D and D″ will be more or less 90^0, so this triangle will be a right-angle triangle. Now if you take a

$$\cos 45^0 = \frac{DD''}{DD'}$$

$DD'' = \cos 45^0 * DD'$

The diagonal BD will elongate and diagonal AC will contract because of the direction of applied shear stress BD will elongate to DD′ were as diagonal AC will shortening into AC′. Now we shall calculated the strain induced on the diagonal BD strain on the BD tensile in nature. Strain is change in length of the diagonal BD will be D′D″

Strain on BD = $= \frac{DD''}{BD} = \frac{\cos 45^0 * DD'}{BD}$

Now let us consider this triangle A, B and D now this is the diagonal of the cube so this angle will be 45^0

$$\sin 45^0 = \frac{AD}{BD}$$

$$BD = \frac{AD}{\sin 45^0}$$

So we will substitute the value of BD as AD

$$BD = \frac{\cos 45^0 * DD'}{BD} = \frac{\cos 45^0 * DD'}{\frac{AD}{\sin 45^0}}$$

$$BD = \frac{1}{2} * \frac{DD'}{AD}$$

Strain on BD = $\frac{\theta}{2}$

Now we calculated the tensile strain induce on BD to induce tensile stress along BD because BD is elongating so it will be tensile stress on BD will be given by

Strain on BD due to tensile stress on BD = $\frac{q}{E}$

Strain on BD due to compressive stress on AC if you see this picture the diagonal AC is getting compress to AC′ so because of the compressive stress on AC there will be some amount of strain along BD that strain will be given by

Strain on BD due to compressive stress on AC = $-\frac{1}{m} * \frac{q}{E}$

This is lateral strain so poissons ratio times stress by young's modulus since it is compressive in nature we will have a minus here. Therefore the total strain on BD

Total strain on BD = $\frac{q}{E} - \left(-\frac{1}{m} * \frac{q}{E}\right) = \frac{q}{E} + \frac{q}{mE} = \frac{q}{E}\left(1 + \frac{1}{m}\right) = \frac{\theta}{2}$

Now N = $\dfrac{q}{\theta}$

$\theta = \dfrac{q}{N}$ substitute value on θ here

$$\frac{q}{E}\left(1+\frac{1}{m}\right)=\frac{\theta}{2}=\frac{q}{N}$$

From this we can get $E = 2N\left(1+\dfrac{1}{m}\right)$

So this the relationship between the rigidity modulus N and young's modulus E.

Relationship Between the Three Elastic Constants (E, K, & N):

Now we see the relation between the three constants of elasticity E, K, and N now we have derived E as

$E = 2N\left(1+\dfrac{1}{m}\right)$ or $\dfrac{1}{m}=\dfrac{E}{2N}-1$ equation (1)

Now and young's modulus of elasticity in term of bulk modulus will be

$$E = 3K\left(1-\frac{2}{m}\right)$$

In the same manner you can simplify

$\dfrac{2}{m}=1-\dfrac{E}{3K}$ or $\dfrac{1}{m}=\dfrac{1}{2}-\dfrac{E}{6K}$ equation (2)

On left hand side we have these two equations $\dfrac{1}{m}$ therefore

$$\frac{E}{2N}-1=\frac{1}{2}-\frac{E}{6K}$$

Simplify and rearrange this equation getting

$$E = \frac{9KN}{N + 3K}$$

This is the relationship between the three elastic constant namely young's modulus E shear modulus k and rigidity modulus N

$$E = \frac{9KN}{N + 3K}$$