B. ARCHITECTURE

MECHANICS OF STRUCTURES – 1 (AR6201)

ELASTIC CONSTANTS

LECTURE -12

Volumetric Strain of a Rectangular bar Subjected to Three Mutually Perpendicular stresses (tensile):

Volumetric strain of a rectangular bar subjected to three mutually perpendicular tensile stresses. p_x is a tensile stress acting along the x-axis which is the length of the bar. p_y is a tensile stress acting along y finally p_z is the tensile stress acting in z direction that is a long is z axis. If you see all the three stresses are tensile in nature. .

p_x - is a tensile stress along x-axis

p_y - is a tensile stress along y-axis

p_z - is a tensile stress along z- axis

Now we shall calculate the strain induced along the three directions, the induced strain along x axis will be given by

$$\varepsilon_x = \frac{p_x}{E} - \frac{p_y}{mE} - \frac{p_z}{mE}$$

Where p_x is the stress or tensile stress acting along x direction and E is the young's modulus of elasticity.

$$p_y = \frac{1}{m} * \frac{p_y}{E}$$

Where $\frac{p_y}{E}$ is the strain acting along y axis which is perpendicular to the direction of x axis, here we are calculating the total strain along x-axis which is longitudinal axis, to get this strain along x-axis, this strain along y axis has to be multiplied poison's ratio, so that's why here we have the term p_y divided by mE. Negative sign indicates that because of the applied force p_x the y direction will undergo contraction then again we have minus p_z divided by mE

In the same manner if we calculate strain along y axis ε_x will be given by

$$\varepsilon_y = \frac{p_y}{E} - \frac{p_x}{mE} - \frac{p_z}{mE}$$

Likewise the strain along z axis is

$$\varepsilon_z = \frac{p_z}{E} - \frac{p_x}{E} - \frac{p_y}{E}$$

Remember in about 3 equations that is equation for ε_x, ε_y, ε_z we should put positive sign for tensile stress and negative sign for compressive stress. If for example if p_x and p_y are compressive stress in nature and p_z is alone tensile in nature then p_z alone will be coming as positive here and the p_x should be should be substituted as $-p_x$ here and p_y should be substituted as $-p_y$. p_x and p_y are compressive in nature

Volumetric strain is given as

$$\frac{\delta\upsilon}{V} = \varepsilon_\upsilon = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

So this is the procedure for calculating volumetric strain of an object which is subjected to stresses along three mutually perpendicular directions.

Definition of Bulk Modulus:

Bulk modulus is indicated by the alphabet capital K, when a body is subjected to three mutually perpendicular stresses of equal intensity ratio of direct stress to the corresponding volumetric strain is known as bulk modulus K will be direct stress divided by volumetric strain.

$$K = \frac{\text{DirectStress}}{\text{Volumetric Strain}}$$

Let us consider an object in the form of a cube. If this cube is subjected to stresses in three mutually perpendicular directions and if all the stresses acting in the three directions are same are of same magnitude p so in this picture this cube is subjected to tensile stress intensity of p in all the three directions x, y and z. For this case the bulk modulus K will be the ratio of direct stress p to the volumetric strain. So p divide by δv by v will be the bulk modulus of elasticity.

Now we shall see the relationship between bulk modulus and young's modulus of elasticity. Once again we are going to consider the same cube so this cube is subjected to three mutually perpendicular tensile stresses of same magnitude along with three axis along the direction of x, y and z axis. Let the side dimension of the cube be l, now we will name the corners of the cube as A, B, C and D and other 4 corners be E, F, G and H.

Now let us calculate the tensile strain experienced or the tensile strain along the side A, B of this particular cube. If you see A, B it oriented towards x axis. So strain along x axis is given by generally strain along x-axis will be taken as longitudinal strain which will be the ratio of change in length by original length which will be equal to direct stress acting along x axis p divided by E young's modulus of elasticity minus the strain induced in y axis the strain induced in y axis is p divided by mE minus the strain induced along z axis which is once again p divide by mE.

$$\varepsilon_x = \frac{\delta L}{L} = \frac{p}{E} - \frac{p}{mE} - \frac{p}{mE}$$

$$\frac{\delta L}{L} = \frac{p}{E}\left[1 - \frac{2}{m}\right]$$

Now the original volume of the cube will be V = l^3

Differentiating we get

$$\delta \upsilon = 3l^2 \delta l$$

From the above equation

$$\delta L = L * \frac{p}{E}\left[1 - \frac{2}{m}\right]$$

So substituting this value δL in the above equation were L=l we get

$$\delta \upsilon = 3l^3 * 1 * \frac{p}{E}\left[1 - \frac{2}{m}\right]$$

$$\delta \upsilon = 3l^3 * \frac{p}{E}\left[1 - \frac{2}{m}\right]$$

Finding the value of E from the above equation

$$E = \frac{3l^3}{\delta \upsilon} * p\left[1 - \frac{2}{m}\right]$$

Where v= l^3, so the equation becomes

$$E = \frac{3 * V}{\delta \upsilon} * p\left[1 - \frac{2}{m}\right]$$

$$E = 3 * \frac{p}{\frac{\delta\upsilon}{\upsilon}} \left[1 - \frac{2}{m} \right]$$

We know that the bulk modulus

K = $\frac{p}{\frac{\delta\upsilon}{\upsilon}}$

So E = $3K\left[1 - \frac{2}{m} \right]$

So this is the relationship between young modulus velocity and bulk modulus.

Example Problem:

A steel block 20 cm x 2cm x 2cm is subjected to a tensile force of 40KN in the direction of its length. Determine the change in volume if E = $2.05*10^5$ N/mm^2. Take 1/m which is poisson's ratio equal 0.3

So we have a steel block which is in the form of rectangular block length of this steel block is 20cm the other two dimension breath 2cm, thickness is also 2cm, it is subjected to a tensile force of magnitude 40km

You have asked to determine the change in volume, the change in volume is $\delta\upsilon$. We know the expression for determining volumetric strain of an object subjected to force along one direction

Volumetric strain is

$$\frac{\delta\upsilon}{V} = \frac{p}{E}\left[1 - \frac{2}{m} \right]$$

p is stress and E is young's modulus.

$$\delta\upsilon = V * \frac{p}{E}\left[1 - \frac{2}{m}\right]$$

Where $\delta\upsilon$ is change in volume, v is the original volume

Now the unit for young's modulus is in Newton per millimetre so we have to convert the applied Force 40Km into newton and also all these dimensions should be converted into millimetres so volume of the bar will be

V = 200x20x20

P = Load / Crossection area perpendicular to the direction of the force

$$= \frac{40 * 10^3}{20 * 20}$$

$$\delta\upsilon = (200 * 20 * 20) * \frac{40 * 10^3}{20 * 20}[1 - 0.6]$$

Finally $\Delta V = 15.616 mm^3$.