B. ARCHITECTURE

MECHANICS OF STRUCTURES – 1 (AR6201)

ELASTIC PROPERTIES OF SOLIDS

LECTURE -11

Definition for Longitudinal Strain & Lateral Strain & Poisson's Ratio:

Longitudinal strain

We shall see the definition for longitudinal strain, now this can be explained with the help of a figure shown below let consider a circular bar, let l be the length of the circular bar and its diameter d. So this circular bar is subjected to a tensile force of magnitude P, so this circular bar will undergo elongation after the application of the tensile force now the length will be increased to L plus additional length δL. Now if you consider the lateral dimension which is the diameter the diameter of the bar will decrease, so the final diameter after a elongation will be d minus the decrease in diameter δd. So after the application of this tensile force the length with elongated get by an amount of δ Land diameter will decrease by amount of δd.

Now we shall see the definition for longitudinal strain it is the deformation per unit length in the direction of the applied force, deformation for unit length is nothing but change in length δL divided by original length L of the bar upon the application of the force speed, which is in the direction of force speed also, so the deformation per unit length which occurs along the direction of the applied force is known as longitudinal strain or it can also be called as primary strain or sometimes we can also call it as linear strain so it is change in length divided by the original length of the bar along the direction of the applied force, since it is a strain it does not have any unit.

Now let's see the definition for lateral strain it is the strain which occurs in every direction at right angles to the direction of the longitudinal strain, now for the same steel bar the lateral strain s is calculated as

Lateral Strain = $\frac{\delta d}{d}$

Where δd – decreases in diameter or change in diameter

d - Original diameter

So this lateral strain occurs at the other direction which is at right angles to the direction of the longitudinal strain, so this is a bar this is the direction of the applied force, longitudinal strain occurs along the direction of the applied force that is in x direction where as the lateral strain occurs perpendicular to the direction of longitudinal strain, so lateral strain occurs perpendicular to x which is the direction of application of force so lateral strain occurs perpendicular to x-axis which is y axis, it also has no unit.

Now let see the definition for poisson's ratio within elastic limit the ratio of lateral strain to longitudinal strain will be a constant known as poison's ratio. So the ratio of lateral strain to the longitudinal strain with an elastic limit will be a constant this constant is known as poisons ratio. Generally for metals poison's ratio varies between 25 and 0.42. For rubber which is a pure elastic material poison's ratio varies between.0.45 and 0.5. Rubber has higher poison's ratio when compared to metals because it will undergo large amount of lateral strain upon the application of axial force.

Example Problem – 1:

Now let's see one problem a steel bar 160 centimetre long is acted upon by forces as shown in the figure below

Find the elongation of the bar.

Take E = 2.1 *10^6 Kg/cm^2.

Now here a steel bar is shown it does not have uniform diameter throughout its length. The bar has different diameters at different segments are different positions also if you see this figure the bar is not only subjected to forces at there ends in addition to these two forces at the ends of the bar we have forces acting on intermediate locations of the bar, so this 4T force and 6T force are acting at intermediate section B and C of this steel bar.

Now we are asked to determine the total elongation of this bar for this we are going to apply a principle known as principal of superposition. We are going to divide this given bar into segments of different lengths that is we are going to divide segment A, B segment B, C and segment C, D separately so each and every divide segment will be considered as a separate free body. The forces will be split up and then the final deformation will be the algebraic sum of deformations of all the split up segments.

Now we shall split the bar into different segments so this is segment A B, now this segment A, B has a diameter of 3 cm, so basically this bar is a circular bar so this is the end A and this is the end B of this particular segment. At the end A which is the free end we have a force of 9T acting towards left. So for equilibrium of this particular portion we should havea force of 9T towards right, so this 9 ton act towards left and this 9 ton act towards right. Now this bar will be under equilibrium.

Now let us consider the mid segment which is segment B, C, so this is the section B and this is the section C or end B and end C. Now if you see this figure at this intermediate section B we have a force of 4T acting towards right. Already for the segment A, B at the end B we have a force of 9 ton acting towards right, so to get a force of 4T acting towards right at this particular segment for the segment B, C at the end B we should apply a force of 5T towards left, so if a super impose these two portions that is portion A, B and B, C if we join these two portions what'll happen? at the junction B we will have a net force of this is 9T towards right this forces 5T towards left, so we'll have net force of 4T acting towards right, so this force will be 5T towards left at the end B to compensate this for equilibrium at the end C we should apply a 5T force towards right.

Now let us come to the final segment which is segment C, D. At end D we have a force of 11T so for equilibrium of this particular segment C, D at the end C there should be a force of 11 ton towards left, now if a super impose B, C and C, D that is if you join B, C and C, D at the junction C what'll happen? this two forces 5T force and 11T force will overlap with one another, 5T force is acting towards right 11T force is acting towards left, so the net force would be that is net force at C would be 6T force acting towards left

Now we have divided the given bar A, B, C, D into segments of three different segments and the forces are split up and written for each and every segments now the total elongation of the bar will be the algebraic sum of elongation of the individuals segments, segment B C has a diameter of 3.5 cm and final segment CD has a diameter of 3 cm. We know the lengths of the individual segments also from the figure, now we shall do the calculation

Total elongation of the bar

$$\Delta L = \frac{P_1 L_1}{A_1 E} + \frac{P_2 L_2}{A_2 E} + \frac{P_3 L_3}{A_3 E}$$

Where P_1 is the force acting on 1st portion, P_2 is the force acting on 2nd portion and P_3 is the force acting on 3rd portion, L_1, L_2, L_3 are the lengths of the portion first second and third potions respectively, similarly A_1, A_2, A_3 are cross section area of first second and third portions, E is the young's modulus of elasticity of the steel bar which is a constant for the three portions.

Now forces acting on this individual split up portion, for the first portion A B we have a tensile force of magnitude 9T we shall the force in terms of kg because young's modulus of elasticity is given in Kg cm^2, so the first before 9T will be 9000 kg multiplied by length of the first portion which is 50cm divided by area of the first portion, diameter of the first portion A, B is 3cm so the area will be π by 4 multiplied by 3 square, E is constant for all the 3 portions so we shall take E outside. P_2 is tensile force 5 ton is 5000 kg multiplied by length of the portion second portion 60cm divided by area π by 4 into 3.5 square, load coming for the third person is also 11 ton tensile force 11000 kg multiplied by length of the portion is 50 cm divided by area is π by 4 multiplied by 3 square. So E is taken outside the square bracket 1hich will be 1 by E.

$$\Delta L = \frac{1}{E}\left[\frac{9000*50}{\frac{\pi}{4}*(3)^2} + \frac{5000*60}{\frac{\pi}{4}*(3.5)^2} + \frac{11000*50}{\frac{\pi}{4}*(3)^2}\right]$$

E = 2.1 * 10^6

$$\Delta L = \frac{1}{2.1*10^6}\left[\frac{9000*50}{\frac{\pi}{4}*(3)^2} + \frac{5000*60}{\frac{\pi}{4}*(3.5)^2} + \frac{11000*50}{\frac{\pi}{4}*(3)^2}\right]$$

$\Delta L =$ 0.0822 cm

So this the total elongation of the bar A, B, C,D here positive sign is coming for all the deformations in the three potions because all the three portions are subjected to tensile force if any one of the force is compressive in nature then we should put negative sign instead of positive sign.

Example Problem – 2:

Now we shall see another problem a metal bar 5cm x 5cm in section is subjected to an axial compressive load of 400 KN. The contraction of a 20cm gauge length is found to be 0.5 mm and the increase in thicknesses 0.04 mm find the value of Young's modulus and poisson’s ratio.

So we have a metal bar which is having a square section which is subjected to axial compressive forces 400 KN, length of the bar is 20cm in length and breadth of the bar is 5cm which is b and finally the thickness is also 5 cm which is t, upon the application of the compression force decrease in length is 0.5 mm and the increase in thickness is 0.04 mm. We have to find out the value of young’s modulus E and the poisons ratio 1 by m.

Now we shall calculate young’s modulus it is the ratio of axial stress to longitudinal strain, axial stress is the applied compressive force which is P divided by the cross section area, cross section area is breath multiplied by thickness, longitudinal strain longitudinal strain is change in length divided by original length.

$$E = \frac{P/bt}{\Delta L/L}$$

$$E = \frac{PL}{(bt)*\Delta L}$$

$$E = \frac{400*10^3*200}{(50*50)*0.5}$$

$$E = 64*10^3 N/mm^2$$

Now we shall calculate poisson’s ratio it is the ration of lateral strain by longitudinal strain.

$$\frac{1}{m} = \frac{\delta d / d}{\delta L / L}$$

It is not a circular bar instead it is a rectangular bar $\delta d / d$ is replaced by $\delta t / t$

$$\frac{1}{m} = \frac{0.04 / 50}{0.5 / 200}$$

$$\frac{1}{m} = 0.32$$

Volumetric Strain (ε_{υ})

Now let us see the definition for volumetric strain it is denoted by the symbol ε_{υ}, volumetric strain is ratio of change in volume which is δv to the original volume which is v of a body when it is subjected to external force so volumetric strain is change in volume divided by original volume when the material is subjected to some external force, so once again it does not have any unit.

$$\varepsilon_{\upsilon} = \frac{\delta v}{\upsilon}$$

Now we shall see the expression for determining the volumetric strain of a rectangular bar subjected to axial force. Let us consider a rectangular bar of length L and breadth b and thickness is t. Let P be the axial tensile force acting on this rectangular bar, so this axial tensile force along the direction of length which will be taken as x axis, the dimensions b is along y axis and the thickness is along z axis because of the application of this axial force P the length L will elongated by an amount of δL, so L increases by δL then the breath b decreases buy an amount of δb, we have already seen that because of the applied tensile force remember the force is applied along the direction of length so the length will increase the length will start increase the other dimensions b and t breath and thickness will decrease, so decrease in the breath is δb will also decrease by an amount of δt, Now we shall get the value of longitudinal strain longitudinal strain which is denoted as a ε_x because the forces acting along the x-axis so longitudinal strain ε_x will be given by axial stress p divided by young's modulus of elasticity E.

$$\varepsilon_x = \frac{p}{E}$$

p is the stress.

E is the young's modulus

Next we will calculate the lateral strain along the y direction that is along y axis ε_y. Lateral strain taking place along y axis will be change in breath divided by original breath.

$$\varepsilon_y = \frac{\delta b}{b}$$

We know that the ratio of lateral strain and longitudinal strain is poisson's ratio.

$$\frac{1}{m} = \frac{\text{LateralStr ain}}{\text{Longitudin alStrain}}$$

Already we have got the value of longitudinal strain, so longitudinal strain multiplied by poisson's ratio will give you the value of lateral strain.

$$\varepsilon_y = \frac{\delta b}{b} = -\frac{1}{m} * \frac{p}{E}$$

Minus sign indicates δb decrease in breath. Similarly lateral strain along the z axis will be

$$\varepsilon_z = \frac{\delta t}{t} = -\frac{1}{m} * \frac{p}{E}$$

Volumetric strain is,

$$\varepsilon_{\upsilon} = \varepsilon_{x} + \varepsilon_{y} + \varepsilon_{z}$$

$$\varepsilon_{\upsilon} = \frac{p}{E} - \frac{p}{mE} - \frac{p}{mE}$$

$$\varepsilon_{\upsilon} = \frac{\delta v}{\upsilon} = \frac{p}{E}\left(1 - \frac{2}{m}\right)$$