# B. ARCHITECTURE

# MECHANICS OF STRUCTURES – 1 (AR6201)

# ELASTIC PROPERTIES OF SOLIDS

# LECTURE -10

## Section Modulus & Radius of Gyration & Elastic limit of an Object:

Section modulus

It is denoted by the alphabet capital Z, it is the ratio of moment of inertia of the section about the centroidal axis on the extreme fibre distance from its centre of gravity or centroidal axis.

$$Z = \frac{1}{y_{max}}$$

Now let us consider a rectangular section of width b and depth equal to d, now this is the CG of rectangular section, the broken line indicates a horizontal centroidal axis passing through the CG. Now according to the definition section modulus is the ratio of moment of inertia of the given section about centroidal axis to the extreme fibre distance from the centroidal axis here I is the moment of inertia of the section about centroidal axis $y_{max}$ is the extreme fibre distance.

Now for a rectangular section we know the value of

$$I_{xx} = \frac{bd^3}{12}$$

Now with respect to the centroidal XX axis extreme fibres are the top most fibre, so this is the top of fibre and the bottom most fibre, distance of the extreme fibre from the centroidal axis XX axis will be so this will be d/2 and also this distance of bottom most fibre will also be d/2. So for a rectangular section extreme fibre distance $y_{max}$ will be $y_{max}$ = d/2, hence if you calculate the section modulus with respect x axis it will be

$$Z_{XX} = \frac{I_{XX}}{y_{max}}$$

$$Z_{xx} = \frac{bd^3}{12} * \frac{2}{d}$$

$$Z_{xx} = \frac{bd^2}{6}$$

In the same manner if we calculate $Z_{yy}$ it will be

$$Z_{xx} = \frac{bd^2}{6}$$

Likewise we can calculate section modulus for square and also circular section.

Radius of gyration (r)

Radius of gyration is indicated by the alphabet r, it is given by this equation

$$r = \sqrt{\frac{I}{A}}$$

Where I is the moment of inertia of the section about its centroidal axis and A is area of the section, its unit will be in centimetre, metre or millimetre.

Elastic limit of an object

Now we shall see the meaning of elastic limit of object, when a body is subjected to an external Force It undergoes some deformation but the molecules of the body offers some resistance against this deformation, once this external force is removed the body will come back to its original position but this is possible if the force or stress which is causing the deformation is only within certain limit, this limit is known as elastic limit.

So naturally if the load or stress exceeds elastic limit then the body will undergo permanent deformation, the material cannot withstand that much resistance to bring back to its original shape on the other hand if the applied load or stress is within elastic limit the material can regain or it can come back to its original shape. So once the material is loaded beyond elastic limit there will be permanent deformation, so it will become plastic material on the other hand is the material is loaded within elastic limit it will have the full elastic property, it will regain its original shape and form.

## Definition of Stress & Strain & Hooke's Law:

Stress (p):

Now we shall see the definition of stress. In most of the Engineering books stress will denoted by the alphabet p, it is the ratio of the applied force to the cross section area of an object or we can also call it as the ratio of the resistance offered per unit cross section area.

Where p is applied force or it will be equal to the internal resistance. In SI system unit of stress will be in $N/mm^2$. Stress can be the compressive in nature of tensile in nature depending upon the nature of applied force.

Now consider a bar like this if it is acted upon by pulling force or a tensile force then the induced stress will be tensile stress, if the same object is acted upon by a pushing force or a compressive force like this then the stress induced will be compressive in nature. So depending upon the nature of force stress can be the compressive or tensile.

Strain (ε):

Strain is indicated by the symbol ε, it is the ratio of change in length or deformation to the original length of the object.

$$\varepsilon = \frac{\Delta L}{L_0}$$

Where $\Delta L$ is deformation or change in length, $L_0$ is the original length of specimen, so it is a unit less quantity.

Hooke's law

When a material is loaded with an elastic limit, the stress is directly proportional to the strain induced, this is a statement for Hooke's law.

$P \alpha \varepsilon$

$$\frac{P}{\varepsilon} = E \text{ (Constant)}$$

Where E is called as modulus of elasticity or young's modulus of the material.

$$\frac{P}{\varepsilon} = E$$

$$\frac{P}{\frac{\Delta L}{L_0}} = E$$

$$\frac{\frac{P}{A}}{\frac{\Delta L}{L_0}} = E$$

$$\Delta L = \frac{PL_0}{AE}$$

Where P is the applied force, $L_0$ is the original length of the specimen, A is the cross section area of the specimen and E is the modulus of elasticity of the given material. Mostly we will be using this equation for calculating deformation in a elastic material.

## Stress-Strain plot for Mild Steel & Concrete:

Stress-Strain plot for mild steel

Now we shall see the stress-strain plot for mild steel. Stress-Strain plot for a mild steel specimen is obtained by conducting a tensile test on steel rod on a mild steel rod using universal testing machine. In the universal testing machine the steel rod will be mounted and then axial tensile force will be applied to the steel rod specimen, now if the axial tensile force is increased if it is increased then it will undergo deformation, so they will be measuring the applied axial force Vs the corresponding deformation. The applied stress will be plotted in y axis, stress is nothing but load by area, tensile force by area of the steel rod specimen and strain will be plotted in x axis which is change in length divided by original length, original length will be the gauge length, the plot will be like this

Now the salient points of the stress- strain plot are shown as A B C D M and F. Now the point A is known as limit of proportionality. Points B and C are known as yield points.In between the portion CD the material will be behaving in a ductile manner so we can call the portion CD to be ductile stage. Point M is knows as ultimate stress point or maximum stress point. Point F is known as breaking point or you can also call it as failure point.

Now if you see the stress-strain plot initially in the portion OA the variation between stress and strain or stress-strain relationship will be linear, so OA will be a straight line. Point A is known as limit of proportionality, elastic limit will be slightly above point A, but for most of the materials elastic limit and limit of proportionality will coincide. So elastic limit will be almost same as limit of proportionality, so it will coincide with point A. Young's modulus of elasticity of any material is the ratio of stress to strain, so for mild steel specimen if you want to get the modulus of elasticity we will be finding out the slope of this particular portion OA, so this is dy, dy will be the stress value with p and this is dx, dx will be strain, so the ratio of stress to strain will give you the young's modulus of elasticity. This ratio should be obtained in the portion OA in order to obtain the young's modulus of elasticity of the steel rod.

Now points B and C are known as yield points. B is upper yield point and C is known as lower yield point. If you see the curve between B and C there will be large amount of deformation taking place even without any stress increase that is even without any load increment between B and C the material starts to yield at rapid rate deformation will be very large. Portion CD is known as a ductile portion or ductile stage in which the material will behave like a ductile material that is the area of cross section of the steel rod will start to reduce along with this deformation. Point M refers to ultimate stress. So at this particular stage this steel as reached is ultimate strength. Beyond M if you see deformation will take place even if the load is decreased or even if the stress is decreased, that is the stress will start to decrease from M it will start to decrease like this, but the deformation or elongation will be keeping on increasing. Finally at the stress corresponding to point F, the material will fail or material will break, so from M to F there will be formation of neck. So if this is a steel rod necK formation you can observe the formation of neck or waist. So between M and F you can observe the formation of neck and finally at point F the material will fail.

Now if you draw the stress-strain plot for a high strength steel it be almost like this except for high strength steel you won't observe well defined failure points like B and C, so for a high strength steel well defined yield points will not be there for high strength steel.

Stress-Strain plot for concrete

Now we shall see stress-strain plot for a concrete. Now for obtaining the stress-strain plot for concrete first of all we should form a concrete cube of dimensions 15cm x 15cm by 15cm in a mould after doing the mix proportion the concrete should be casted in a mould of dimensions 15 by 15 by 15 cm, the concrete should be cured for a period of 28 days by immersing the mould along with the concrete in water. After a period of 28 days off curing, this concrete specimen will be tested in a compression testing machine, steel rod will be tested in the tensile testing machine or UTM here it will be tested in a compression testing machine, so in a compression testing machine axial compressive force will be applied, this axial compressive force will be kept on increasing and then the deformation will be measured using the value of applied axial compressive force andthe deformation measured stress-strain plot will be plotted. Stress p we will be taken in y axis and strain will be plotted in x-axis.

Now if you see this shape of the stress-strain plot it'll be almost curvilinear except for the initial portion AB, so the stress-strain plot will be a straight line for the initial small portion AB beyond point B the relationship between the stress p and strain ε will not be a linear one it will be a curvilinear one, this is due to the fact that there will be formation of fine cracks within the concrete cube, so beyond point B there will be formation of fine fractures or cracks within the concrete cube and hence the relationship between stress and strain won't be linear one, so AB will be the linear portion a small linear portion M is the maximum stress point and F is the failure point , this maximum stress for most of the concrete specimen will occur at strain of 0.2 %. 0.2% will be 0.002 and the failure will occur at a strain ranging between 0.003 to 0.004. So failure point will be attained at a strain rate varying between 0.3% to 0.4%. So stress corresponding to point M is maximum compressive stress so this $P_{max}$ is maximum compressive stressor this will be called as ultimate compressive strength of the concrete cube ultimate compressive strength of concrete cube, likewise for the steel rod also the stress corresponding to the point M will be the maximum tensile stress which will be the ultimate tensile strength ultimate tensile strength of the Steel rod. So $P_{max}$ here for the concrete cube is the maximum compressive stress or ultimate compressive strength of concrete. In order to get the young'smodulus of elasticity of the concrete we should take the slope of the curve within the portion of 0.45 $P_{max}$, so this point will correspond to 0.45 of the $P_{max}$, so young's modulus of elasticity is the ratio of stress to strain within this particular portion, so if you take this as X we should take the slope of this particular portion between A and X , so this is stress and this is strain, So stress divide by the strain will be the young's modulus of elasticity of the concrete cube.

Problem:

Now we shall see an example problem for calculating stress, strain and deformation. A mild steel rod is 25 mm in diameter and 200 cm long the rod is subjected to axial pull of 50 KN. Find

1. The intensity of stress
2. Strain
3. Elongation

Take E = $2*10^5$ N/mm$^2$.

Now first of all we shall calculate the value of stress. Now the mild steel rod is subjected to axial pull hence the nature of stress will be tensile, stress is denoted by the alphabet p which is

(i) Stress (tensile) P = $\frac{P}{A}$

Where 1KN = 1000N = $10^3$ N, Dia = 25mm, L=200cm

Stress (tensile) P = $$\frac{50*10^3(N)}{\frac{\pi}{4}(25)^2(mm)^2}$$

Stress (tensile) $P = 101.86\ N/mm^2$.

(ii) Strain $\varepsilon = \dfrac{P}{E}$

Strain $\varepsilon = \dfrac{101.86 N/mm^2}{2 * 10^5 N/mm^2}$

Strain $\varepsilon = 5.09*10^{-4}$.

(iii) Elongation $\Delta L = \varepsilon * L_0$

Elongation $\Delta L = 5.09 * 10^{-4} * 2000$

Elongation $\Delta L = 1.02$ mm.