Mechanics of Structure – I

Lecture 1

Force is an agent which produces or tends to produce, destroys or tends to destroy a motion. SI units of force is Newton. Basically force is a vector quantity. So it has both magnitude as well as direction.

Now we shall see some in-depth meaning of force. We shall assume that this object is at rest. Now if you apply a force to this object then that force causes the object to move. On the other hand if you have a moving object and if you give a force to it, then that object can be brought to standstill. Hence force is an agent which produces or tends to produce or destroys or tends to destroy a motion. As we have seen earlier, the SI unit for force is Newton. We know that force is mass times acceleration. If you have an object of mass 1kg and if that object is moving with an acceleration of $1m/sec^2$ then we have force as 1N which means that if you have an object of mass equal to 1kg and if that object moves with an acceleration of $1m/sec^2$ and if you apply a force of 1N to that object then we can bring that object into stand still position. So hence newton is the force which is required to bring down to an object having a mass of 1Kg into stand still position if the object moves with the acceleration of $1m/sec^2$ or in other way if you apply a force of 1N to an object having a mass of 1Kg then that object move with the acceleration of $1m/sec^2$ so therefore N will be equal to Kgm/sec^2. Let us consider an object whose mass is 1kg, if you drop this object having a mass of 1kg then it will be falling towards the centre of the earth with an acceleration of $9.81m/sec^2$ now we shall calculate the force that this object will be exerting. Force will be equal to mass times acceleration and hence mass of the object is 1kg multiplied by acceleration due to gravity, if the object is dropped down, it will be falling towards the earth under the influence of gravity. Hence the acceleration will be because of gravity. Acceleration due to gravity under normal circumstances is $9.81m/sec^2$. Now you have force will be equal to $9.81Kgm/sec^2$. Already we have seen that $1N=1kgm/sec^2$ therefore $9.81Kgm/sec^2$ will be equal to 9.81N. So if you

have an object having a mass of 1kg it will exert an equivalent force of 9.81N. Therefore we can say that 1Kg will exert a force of 9.81N i.e. 1Kgf=9.81N. So for solving problems, often we will be using this for conversion i.e. for converting mass from Kg into Newton we will be using this conversion. If several forces are acting on object then we call it as system of forces or we call it as force systems. If an object is subjected to several forces then it is called as force systems. Consider an object like this, if it is acted upon by several forces then we say that this object is subjected to system of forces. Now we shall see the several types of forces system. Force system is broadly classified into two types. Coplanar and Non-Coplanar force systems. Coplanar force system is one in which all the forces will be acting on the same plane, so several forces acting on the same plane will be called as coplanar force system. On the other hand if the forces are acting on different planes it will be called as non-coplanar force system. We are going to concentrate only about coplanar force system. Coplanar force systems are further classified into four types. The 1^{st} one is collinear force system. In coplanar collinear force system all the forces will be acting on a common plane and all the forces will have a common line of action. So here we have 3 forces F_1, F_2, and F_3 and all the three forces are having a common line of action so this type of force system is known as collinear force system. Now let us come to Concurrent force system. In case of Coplanar concurrent force systems all the several forces will be once again acting in the same plane and all the forces will have a common point of origin. Here once again we have 3 forces F_1, F_2, and F_3 and all the 3 forces are originating from a common point O. Now the forces can originate from a common point like this or the forces can converge towards the common point. Such a kind of force system is known as concurrent force system. Next is parallel force system. In case of coplanar parallel force system, all the forces will be acting once again in a same plane and all the forces will be parallel to one another. Here the 3 forces F_1, F_2, and F_3 are parallel to one another. If you see the figure F_1 and F_3 are acting towards right and F_2 is acting towards left. This type of force system is known as unlike parallel force system. If all the forces are acting along the same direction and if all the forces are parallel to each other then such kind of force

system is known as like parallel force system. So parallel force system is of two types unlike parallel force system and like parallel force system. Finally you have non-concurrent and non-parallel force system in which the forces will be neither parallel to each other nor they will be converging or diverging towards common point. So coplanar non concurrent non parallel force system all the forces will be acting in same plane but they won't be having a common intersection point and they also won't be parallel to one another like this. In this picture you can see that the three forces are neither parallel to each other nor they have a common intersection point.

Now we shall see about resultant of several forces. Incase if a body is subjected to several forces then we can find resultant for those several forces. Resultant of forces is a single force that effectively replaces all other forces acting on the object without changing the external effects on it. Suppose you have a rectangular object. So this rectangular object is acted upon by 2 forces. 10N force pushes the object towards right and 5N force acts in the other direction and it pushes the object towards left. These two forces can be replaced by a single force known as resultant and that resultant should produce same effect as that of the two forces which are producing on this object. Hence the resultant would be like this. The resultant of these two forces will be 5N force which will be pushing the object towards left. If you see these two cases the net effect on the object will be same. In the first case you have two forces acting on the object producing a certain effect and in the second case we have replaced the two forces by a single force known as resultant. This resultant will also produce the same effect as that of the two forces which were acting in case 1. Now we shall see how to resolve the given force. As we discussed earlier, the force is a vector quantity and hence it has got both magnitude as well as direction. A force weill be represented by a line the length of the line to some scale will represent its magnitude and also this line will have an arrow mark. This arrow mark indicates the direction of the application of the force. This is how we represent a force. Resolution of force is finding the components of a given force along two directions namely vertical and horizontal directions. Suppose

you have inclined force acting like this. Resolution is nothing but finding the components of this inclined force along two directions namely the horizontal direction which is X axis and the vertical direction which is Y axis. Let the inclination of force with the horizontal be θ. Vertical component of this force is represented by V and the horizontal component of the force along X direction is represented as H. So vertical component is the component of the force along Y axis or vertical direction and horizontal component of this force F will be along X axis and that component is denoted as H. Now to find out the vertical component let us consider this right angled triangle. This particular right angled triangle, in this right angled triangle this 90^0. Now for this right angled triangle we shall take

$$\sin\theta = \frac{V}{F}$$

So therefore the vertical component will be given by V=Fsinθ. And in the same right angled triangle, if we take cosθ,

$$\cos\theta = \frac{H}{F}$$

so therefore the horizontal component will be given by H=Fcosθ. Now if you see the direction of F, the force acts from this point that is origin and it ends up here. So the direction of horizontal component should be towards right and the direction of vertical component should be in the upward direction. So to reach from this particular point to this particular point we have to move horizontally and then we have to move vertically. So the direction of horizontal component will be on the positive X direction and the direction of vertical component will be on the positive Y direction that is on the upward direction. Now we have found out the magnitude of vertical component along with its direction as well as the magnitude of horizontal component along with its direction.

Now we shall see a problem. We shall workout a problem then we will all be having a clear picture about how to resolve a given force and how to find the resultant of force system. A particle is acted upon simultaneously by the following forces. The 1^{st} force is 200N and is

inclined 30^0 to the north of east, the 2^{nd} force is 250N acting towards the north, the 3^{rd} force is 300N towards North West and finally the 4^{th} force is 350N inclined at 40^0 to the south of west. Find the magnitude and direction of the resultant.

So first of all we shall mark the four directions, north, south, east and west. The magnitude of 1^{st} force is 200N and is inclined 30^0 to the north of east, so the 1^{st} force is 200N and is inclined 30^0 to the north of east, so from east it is acting at an angle of 30^0 towards north. This is the 1^{st} force we shall name this force as F_1. The second force is 250N which is acting towards north, the second force actually acts towards north, its magnitude is 250N so this force is represented as F_2. the third force is 300N which is acting towards north west so exactly between north and west we have the third force of magnitude 300N which is F_3 so the inclination of force with respect to west will be 45^0 also the inclination of this force with respect to north will be 45^0 because this force is acting along north west direction. Finally the 4^{th} force is 350N inclined at 40^0 to the south of west. So 40^0 measured from west towards south. The magnitude of this final force is 350N. So in the problem you are asked to determine the magnitude and direction of the resultant of all the four forces. Here the resultant R will be computed as

$$R = \sqrt{\sum H^2 + \sum V^2}$$

So this is the magnitude of the resultant and the direction of the resultant is given as

$$\theta = \tan^{-1}\left(\frac{\sum V}{\sum H}\right)$$

$\sum H$

is the algebraic sum of all horizontal component and

$\sum V$

is the algebraic sum of all vertical component of the forces. Here four forces are acting. We need to find out algebraic sum of all the horizontal

components of the forces as well as the algebraic sum of all the vertical components of the forces. Now let us consider the first force which is the force F_1 this force is inclined at 30^0 to the horizontal we need to find out the horizontal and vertical components of this force F_1. This force F_1 starts at this particular point and it ends here. so from the final ending point let us drop a perpendicular line. So this particular value will be the vertical component of this force F_1 and the horizontal component of this force F_1 will be along the X axis and this will be H_1 so to reach from this particular point and to end up here horizontally we have to go towards right and vertically we have to move up. So the direction of the horizontal component of F_1 will be on the positive X axis and the direction of vertical component of force F_1 will be on the positive Y axis i.e. in the upward direction. Now H_1 will be $F_1\cos30^0$ and V_1 will be equal to $F_1\sin30^0$ now the sign convention for the forces will be like this all the forces going in upward direction will be taken as positive and hence the forces going in downward direction will be taken as negative. Similarly the forces going towards right i.e. along the positive X axis will be taken as positive forces and forces going towards left i.e. on the negative x axis will be taken as –ve. These are the sign conventions which we are going to adopt for our resolution since H_1 acts towards right, sign for H_1 will be +ve and since V_1 is moving upwards sign for V_1 will be +ve. Now in the same manner we shall resolve the second force which is the force F_2. So before that F_1 is 200 so it will be 200cos30, the horizontal component will be +200cos30 and the vertical component of the first force will be +200sin30. Now the second force actually is a vertical force F_2 whose magnitude is 250N. Let H_2 be the horizontal component of this force. So horizontal component of F_2 will be $F_2\cos90$ since the force F_2 is acting in vertical direction θ will be 90^0 so if we substitute θ as 90, H_2 will be $F_2\cos90$ so which will be 0. The vertical component of this force will be $F_2\sin90$ which will be equal to F_2 itself, the sign convention for vertical component will be + since this component acts in the vertical direction. So V_2 will be equal to 250N. Now the third force F_3 is 300N which acts at an inclination of 45^0 from the west direction so the vertical component will be V_3 and the horizontal component of this particular force is H_3, force F_3 starts from

its origin and it ends up at this particular point so to travel from this origin and horizontally we have to move towards left and vertically we have to move in the upward direction now H_3, horizontal component of F_3 will be $F_3\cos45$. Since F_3 goes towards left the sign convention would be –ve so H_3 will be $-F_3\cos45$. Now vertical component V_3 will be $F_3\sin45$. If you see the direction of this vertical component V_3 which is going in the upward direction hence this sign will be +. So V_3 will be $+300\sin45$ and H_3 will be $-300\cos45$. And finally we have one more force which is of magnitude 350N, it is at an inclination of 40^0 from west. Now we shall resolve this force along vertical and horizontal direction so this is the horizontal component H_4 and this is the vertical component V_4. So the force F_4 starts from the origin i.e. origin is this point and it ends up at the tip of the arrow mark to travel from the origin and to end up here, horizontally we have to move towards left and vertically we have to move in the downward direction. Now H_4 will be $F_4\cos40^0$. So it will be $350\cos40$. Since H_4 moves towards left, the sign convention will be –ve and V_4 will be $F_4\sin40$ i.e. $350\sin40$ if you see the direction of V_4 it is moving in the downward direction and hence its sign convention will be -. We shall find out the algebraic sum of horizontal component of all the forces

$$\sum H = +200\cos 30 + 0 - 300\cos 45 - 350\cos 40$$
$$= -307.04N$$

Similarly of we compute the sum of all vertical components of the forces,

$$\sum V = +200\sin 30 + 250 + 300\sin 45 - 350\sin 40$$
$$= -337.16N$$

So the magnitude of the resultant R will be

$$R = \sqrt{\sum H^2 + \sum V^2} = 456\text{N}$$

and the direction of resultant is nothing but inclination θ which is given by

$$\theta = \tan^{-1}\left(\frac{\sum V}{\sum H}\right)$$

Which comes as $47^0 40'36''$. So we have computed the magnitude of the resultant as well as its direction.