Bachelor of Architecture

Mathematics

Lecture 09

In this lecture we are going to see the Reductions formula for trigonometric functions, Taylor's theorem and then Summary.

Reduction formula for trigonometric functions:

We know the trigonometric functions are the functions deals with the sine, Cos, cosine, tan etc. And in that this reductions formula is a special technique of integration which is used for higher power integrand. Here the power of integrand is reduced and the process is continued till we get a power which can be easily integrated.

Reduction formula is a formula which connects a given integral with another integral which is of the same type but of a lower degree or lower order or otherwise easier to evaluate using any technique of integration.

Result - I:

Obtain the reduction formula for $\int \sin^n x dx$.

Here the sin x is the trigonometric function and it is obtained from the basic triangle called right angle triangle. Where the sin is define as the opposite by hypotenuse.And the integration of sin x give –Cos x, then the integration of higher order sin x can be done using this reduction formula.

Let us consider

$$I_n = \int \sin^n x dx$$

$$= \int \sin^{n-1} x Sinx dx$$

This integration is in the form of,

$$\int u dv = uv - \int v du$$

$$= Sin^{n-1}x(-Cosx) - \int (n-1)Sin^{n-2}xCos(-Cosx)dx$$

$$= Sin^{n-1}xCosx + (n-1)\int Sin^{n-2}xCos^{2}xdx$$

$$= -Sin^{n-1}xCosx + (n-1)\int Sin^{n-2}x(1-Sin^{2}x)dx$$

$$= -Sin^{n-1}xCosx + (n-1)(\int Sin^{n-2}xdx - \int Sin^{n}xdx$$

$$= -Sin^{n-1}xCosx + (n-1)(I_{n-2} - I_{n})$$

$$= I_{n} + (n-1)I_{n} = -Sin^{n-1}xCosx + (n-1)I_{n-2}$$

$$I_{n} = \frac{-Sin^{n-1}xCosx}{n} + \frac{(n-1)}{n}I_{n-2}$$

So this is the reduction formula for the given higher order integration. Now it is the case of indefinite integral. If it is considered as definite integral then,

$$I_{n} = \int_{0}^{\frac{\pi}{2}} Sin^{n}xdx$$

$$I_{n} = \left[\frac{-Sin^{n-1}xCosx}{n}\right]_{0}^{\pi/2} + \left(\frac{n-1}{n}\right)I_{n-2}$$

$$I_{n} = \frac{n-1}{n}I_{n-2}$$

Replacing the n terms we get,

$$I_{n-2} = \left(\frac{n-3}{n-2}\right)I_{n-4}$$

$$I_{n-4} = \left(\frac{n-5}{n-4}\right)I_{n-6}$$

$$I_{n-6} = \left(\frac{n-7}{n-6}\right)I_{n-8}$$

Now from this expression two cases arrives, the first case is,

Case 1:

When n is a positive or Odd, Then

$$I_n = \left(\frac{n-1}{n}\right)\left(\frac{n-3}{n-2}\right)\left(\frac{n-5}{n-4}\right)......\frac{2}{3} I_1 ------(1)$$

But we know that,

$$I_n = \int_0^{\pi/2} Sinx dx = (-Cos)_0^{\pi/2} = 1$$

$$I_n = \left(\frac{n-1}{n}\right)\left(\frac{n-3}{n-2}\right)\left(\frac{n-5}{n-4}\right)......\frac{2}{3}$$

Suppose when n is the positive even integer,

$$I_n = \left(\frac{n-1}{n}\right)\left(\frac{n-3}{n-2}\right)\left(\frac{n-5}{n-4}\right)......\frac{1}{2} I_0$$

In this case where I_0 is,

$$I_0 = \int_0^{\frac{\pi}{2}} Sin^0 x dx = (x)_0^{\pi/2} = \frac{\pi}{2}$$

$$I_n = \left(\frac{n-1}{n}\right)\left(\frac{n-3}{n-2}\right)\left(\frac{n-5}{n-4}\right)......\frac{1}{2}\frac{\pi}{2}$$

$\int_0^{\pi/2} Sin^n dx = \left\{\left(\frac{n-1}{n}\right)\left(\frac{n-3}{n-2}\right)\left(\frac{n-5}{n-4}\right).........\frac{2}{3}\right\}$ When n is a positive odd natural number.

$\int_0^{\pi/2} Sin^n dx = \left\{\left(\frac{n-1}{n}\right)\left(\frac{n-3}{n-2}\right)\left(\frac{n-5}{n-4}\right).........\frac{1}{2}.\frac{\pi}{2}\right\}$ When n is a positive even natural number.

Similarly we can obtain the reduction formula for the Cos function and tan function.

$$\int Cos^n x dx = \frac{Cos^{n-1} x Sin}{n} + \frac{(n-1)}{n} I_{n-2}$$

$\int_0^{\frac{\pi}{2}} Cos^n x dx = \left\{\left(\frac{n-1}{n}\right)\left(\frac{n-3}{n-2}\right)\left(\frac{n-5}{n-4}\right).........\frac{2}{3}\right\}$ When n is positive and odd function.

$$\int_0^{\frac{\pi}{2}} Cos^n x dx = \left\{\left(\frac{n-1}{n}\right)\left(\frac{n-3}{n-2}\right)\left(\frac{n-5}{n-4}\right)\ldots\ldots\ldots\frac{1}{2}\frac{\pi}{2}\right\}$$ When n is positive and even function.

Similarly we can obtain the integral function for the tan function also as,

$$I_n = \int \frac{\tan^{n-1} x}{n-1} - I_{n-2} .$$

Example 1:

Evaluate $\int_0^{\pi/2} Sin^{2m} x dx, m \in N$.

Solution:

In this problem the 2m is the higher order.Here 2m is the even positive integer. We know the formula for the positive integer,

$$\int_0^{\pi/2} Sin^{2m} dx = \left\{\left(\frac{2m-1}{2m}\right)\left(\frac{2m-3}{2m-2}\right)\ldots\ldots\ldots\frac{3.1}{4.1}.\frac{\pi}{2}\right\}$$

$$= (2m)(2m-2)\ldots\ldots\ldots 4.2$$

$$\frac{= (2m-1)(2m-2)(2m-3)\ldots.3.2.1}{\left[(2m)(2m-2)\ldots\ldots.4.2\right]^2}.\frac{\pi}{2}$$

$$= \frac{2m!}{2^m m!}.\frac{\pi}{2}$$

Taylor's Theorem with Lagrange form of reminder after n terms:

It is a mean value theorem. If a function f(x) is such that

a) $f(x), f'(x), f''(x)\ldots\ldots\ldots\ldots\ldots f^{n-1}(x)$ are continues in $[a, a+h]$.
b) $f^n(x)$ exist in $[a, a+h]$. that at least one θ between 0 and 1 such that

$$F(a+h) = f(a) + hf'(a) + \frac{h^2}{2!} f''(a) + \ldots\ldots + \frac{h^{n-1}}{(n-1)!} f^{n-1}(a) + \frac{h^n}{n!} f^n(a+\theta h)$$

Where $\frac{h^n}{n!} f^n(a+\theta h)$ is called Lagrange's form of remainder after n terms, i.e., $R_n = \frac{h_n}{n!} f^n(a+\theta h)$.

Summary:

Reduction technique is a special technique to integrate higher power functions. Reduction formula is the one which connects a given integral with another of the same type but of a lower degree using any technique of integration. The Taylor's theorem with Lagange's form of reminder after n terms are also discussed.

After listening to this lecture you can answer the following questions.

Questions:

1. What do you mean by reduction formula?
2. Given the reduction formula for $\int Sin^{n} x$.
3. Define Taylor's theorem.