

## FAQ's

1. What do you mean reduction formula?

**Answer:**

Reduction formula is a one which connects a given integral of some type but of a lower degree of easier to evaluate using any technique of integration.

2. Give the reduction formula for  $\int \sin^n x dx$

**Answer:**

I. If n is positive odd integer,

$$I_n = \left(\frac{n-1}{n}\right)\left(\frac{n-3}{n-2}\right)\left(\frac{n-5}{n-4}\right)\dots\dots\dots \frac{2}{3} I_1$$

II. If n is positive even integer then

$$I_n = \left(\frac{n-1}{n}\right)\left(\frac{n-3}{n-2}\right)\left(\frac{n-5}{n-4}\right)\dots\dots\dots \frac{1}{2} I_0$$

3. Evaluate  $\int_0^{\frac{\pi}{4}} \sec^4 x dx$

**Answer:**

Putting  $n=4$  is the reduction formula for  $\sec^4 x dx$  we get

$$I_4 = \frac{\sec^2 x \tan x}{3} + \frac{2}{3} I_2$$

$$\int_0^{\frac{\pi}{4}} \sec^4 x dx = \left| \frac{\sec^2 x \tan x}{3} \right|_0^{\frac{\pi}{4}} + \frac{2}{3} \int_0^{\frac{\pi}{4}} \sec^2 x dx$$

$$= \frac{2}{3} + \frac{2}{3} \left| \tan x \right|_0^{\frac{\pi}{4}} = \frac{2}{3} (1+1) = \frac{4}{3}$$

4. Evaluate  $\int_0^{\frac{\pi}{2}} \cos^7 x dx$

**Answer:**

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \cos^7 x dx$$

$n=7$  is odd positive integer

$$\begin{aligned} \text{Therefore } I &= \int_0^{\frac{\pi}{2}} \cos^7 x dx = \frac{(7-1)(7-3)(7-5)}{(7)(7-2)(7-4)} \\ &= \frac{6 \times 4 \times 2}{7 \times 5 \times 3} = \frac{16}{35} \end{aligned}$$

5. State Taylor's theorem

**Answer:**

If a function  $f(x)$  is such that  $f(x), f'(x), f''(x), \dots, f^{(n-1)}(x)$  are continuous in  $[a, a+h]$ .

Then at least one value  $\theta$  between 0 and 1 such that

$$f(a+h) = f(a) + f'(a)h + \frac{h^2}{2!} f''(a) + \dots + \frac{h^{n-1}}{(n-1)!} f^{(n-1)}(a) + \frac{h^n}{n!} f^{(n)}(a + \theta h).$$