Bachelor of Architecture

Mathematics

Lecture 8

In this lecture we are going to see about Integration. In that we will discuss integration of rational function, integration of trigonometric function and integration of irrational function. And then we will discuss about the properties of definite integrals and finally summary.

Integration of rational and irrational function:

We know the integration is the concept used to find the length of the line if the starting point and the end point of the line is known and to find the area of a curve i.e., using the double integral function and we can find the volume of a curve using the triple integral function.

Rational function:

A rational function is the one which is the quotient of two polynomials. For example,

$\frac{1}{1-x^2}, \frac{x^3}{1+x^2}, \frac{5x^3-3x^2+15x-\sqrt{2}}{x^5+2x^2+1}$ are all rational functions.

In the definition of rational function we will not see $\sqrt{x}$ neither $\ln(x)$ or $|x|$. A rational function is called proper if the degree of the numerator is less than than the degree of the denominator and improper otherwise. Indefinite integrals of the rational function is accomplished by any of the following method.

1. Polynomial division.
2. Partial function expansion.
3. Term by term integration.

Collectively we shall classify the integration of rational and irrational functions into different types and solve it. Suppose the given rational function is,

Type I:

$$\int \frac{dx}{quadratic} or \int \frac{dx}{\sqrt{quadratic}}$$

$$i.e \int \frac{dx}{ax^2+bx+c} or \int \frac{dx}{\sqrt{ax^2+bx+c}}$$

To do the above integration some steps are involved, they are

Step 1:

Make the co-efficient of x^2 unity by taking its numerical co-efficient outside.

Step 2:

Complete the square in terms of x by adding and subtracting $\left(\frac{co-efficient\,.of\,.x}{2}\right)^2$.

Step 3:

Apply proper standard form of integration.

Type II:

$$\frac{Linear}{quadratic} or \frac{Linear}{\sqrt{quadratic}} or Linear\sqrt{quadratic}$$

$$\int \frac{px+q}{ax^2+bx+c} or \int \frac{px+q}{\sqrt{ax^2+bx+c}} or$$

$$\int (px+q)\sqrt{ax^2+bx+xdx}$$

To solve this integration the following steps are used.

Step 1:

In this we make linear as derivative of quadratic by putting

$$Linear = A\frac{d}{dx}(quadratic)+B.$$

Step 2:

Equate the co-efficient of x and constant term to obtain A and B.

Step 3:

Replace the linear of problem by step (1) and then use the standard integration.

Type III:

$\int \frac{dx}{L\sqrt{L}} or \int L\sqrt{L} or \int \frac{L}{\sqrt{Q}} dx.or \sqrt{L}$ is linear or whenever $\sqrt{L}$ is there then put $L = z^2$. Hence L stands for linear algebraic and Q stands for Quadratic polynomial.

Type IV:

$$\int \frac{dx}{linear\sqrt{quadratic}} \, put \sqrt{quadratic} = z.or.x = \frac{1}{z}$$

Type V:

$$\int \frac{dx}{quadratic\sqrt{quadratic}} \, put...linear = \frac{1}{z}$$

So like this the given rational function can be classified into five types and the steps can be used to integrate the rational function. Now we will apply this concepts in a problem.

Example 1:

Evaluate $\int \frac{dx}{3x^2 + 6x + 5}$.

Solution:

The given integration is,

$$\int \frac{dx}{3x^2 + 6x + 5}$$

First step we need to do is we need to take the co-efficient of the x out,

$$= \frac{1}{3} \int \frac{dx}{x^2 + 2x + \frac{5}{3}}$$

Now taking the denominator,

$x^2+2x+\frac{5}{3}$

$2\,x^2+2x+1+\frac{5}{3}-1$

In the above step we have added 1 and subtracted 1, so that the quadratic form of the given equation will not be changed.

$(x^2+2x+1)+\frac{2}{3}$

$(x+1)^2+\frac{2}{3}$

Substituting these in the equation, we get

$$=\frac{1}{3}\int\frac{dx}{(x+1)^2+\frac{2}{3}}$$

$$=\frac{1}{3}\int\frac{dx}{(x+1)^2+(\sqrt{\frac{2}{3}})^2}$$

So this is in the standard integral of $\int\frac{dx}{x^2+a^2}$, hence

$$=\frac{1}{3}\int\frac{dx}{3x^2+6x+5}$$

$$=\frac{1}{3}\times\frac{\sqrt{3}}{2}\tan^{-1}\left(\frac{x+1}{\frac{\sqrt{2}}{\sqrt{3}}}\right)+C$$

$$=\frac{1}{\sqrt{6}}\times\tan^{-1}\left(\frac{\sqrt{3}}{\sqrt{2}}(x+1)\right)+C$$

Example 2:

Evaluate $\int\frac{(1+\sin x)\cos x}{\sqrt{Sin^2 x-2\sin x-3}}dx$

Solution:

In this problem the trigonometric function is used so by solving this problem we will understand the integration of a trigonometric function.

First substitute $Sinx = z$ and find the value of dx,

$$Sinx = z$$

$$dz = Cosxdx$$

After this the integral will become,

$$\int \frac{1+z}{\sqrt{z^2-2z-3}}dz$$

Now we changed the integral to the form linear over quadratic, so then substitute

$$put..1+z = A\frac{d}{dz}(z^2-2z-3)+B$$

$$1+z = A(2z-2)+B$$

Then equating the co-efficient of z and the constant term we get,

$$A = \frac{1}{2}$$

$$-2A+B=1$$

$$B=2$$

We got the value of A and B after substituting the co-efficient of z and the constant term.

$$\therefore \int \frac{(1+z)dz}{\sqrt{z^2+2z-3}}$$

$$= \int \frac{\frac{1}{2}(2z-2)+2}{\sqrt{z^2-2z-3}}dz$$

$$= \frac{1}{2}\int \frac{2z-2}{\sqrt{z^2-2z-3}} + 2\int \frac{dz}{\sqrt{z^2-2z-3}}$$

$$=\frac{1}{2}\sqrt{z^2-2z-3}+2\int\frac{dz}{(z-1)^2-4}$$

$$=\frac{1}{2}\sqrt{z^2-2z-3}+2\log\left|(z-1)+\sqrt{z^2-2z-3}\right|+C$$

$$=\frac{1}{2}\sqrt{Sin^2x-2Sinx-3}+2\log\left|(Sinx-1)+\sqrt{Sin^2x-2Sinx-3}\right|+C$$

Here in this problem we learned how to integrate a rational function involving trigonometric function.

Properties of definite integrals:

1. $\int_a^b f(x)dx=-\int_a^b f(x)dx$
2. $\int_a^b f(x)dx=\int_a^b f(u)dx$ this is referred as change of variable.
3. $\int_0^a f(x)dx=-\int_0^a f(a-x)dx$
4. $\int_a^b f(x)dx=-\int_a^b f(a+b-x)dx$

Summary:

In this lecture we have learned about the integration of rational and irrational function. The rational function is the one which is the quotient of two polynomials. This rational and irrational function can be integrated using varies integration technique.

Partial fraction expression is one of the techniques to perform integration of rational function. Properties of definite integrals are discussed. After listening to this lecture you can answer the following questions.

Questions:

1. Define rational function.
2. Evaluate $\frac{dx}{x(x^5+1)}$
3. List the properties of definite integrals.