# Bachelor of Architecture

# Mathematics

# Lecture 2

In this lecture we are going to discuss the Exponential function, DE-MOIVER's theorem and then summary.

**Exponential function:**

Exponential function is an algebraic function. It can be expand as series and it is denoted as $e^x$. This is applicable for all real values of x. Suppose any complex function is exponential it is denoted as $e^z$. Where this z is the complex value made up of real and imaginary values. In series form the exponential of the real value x will be,

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots\ldots\ldots\ldots$$

**Example 1:**

Find the co-efficient of $x^n$ in the expansion of $e^{a+bx}$.

**Solution:**

To obtain the solution for this problem, first expand the given function as,

$$e^{a+bx} = e^a . e^{bx}$$

$$= e^a \left[ 1 + bx + \frac{(bx)^2}{2!} + \ldots.. \frac{(bx)^n}{n!} + \ldots..\infty \right]$$

Here in these problem we are asked to find only the co-efficient of $x^n$. So let us take that component alone. Co-efficient of $x^n$ will be,

$$e^a . \frac{b^n}{n!}$$

So in this example we learned how to apply the series form of the exponential function.

**Complex Function:**

As we say earlier z is a complex function and the z can be expressed as

$Z = u + iv$ or $Z = x + iy$

But here in this problem will take z value as,

$Z = x + iy$

$e^z = e^{x+iy}$

$= e^x . e^{iy}$

We know that this $e^x$ can be expressed as,

$$= \left[ 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + ...... + \infty \right] e^{iy}$$

To leave the exponential term we should use a trigonometric function,

$= e^x . (Cosy + iSiny)$

This is how the imaginary part of a function can be represented. This $e^z$ is also a periodic function with periodicity $2\pi i$. So this can be expressed in equation as,

$e^z + 2\pi i = e^z$

$e^z + 2n\pi i = e^z$

One of the standard result, in this exponential function is that,

$$e^x = \left[ 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + ...... + \infty \right]$$

Like these the negative function can be expressed as,

$$e^{-x} = \left[ 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + ...... + \infty \right]$$

The average of these two functions will be,

$$\frac{e^x + e^{-x}}{2} = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + ..... + \infty$$

$$\frac{e^x - e^{-x}}{2} = x + \frac{x^3}{3!} + \frac{x^5}{5!} + ..... + \infty$$

Here if we substitute 1 instead of x we get,

$$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + ...... + \infty$$

$$e^{-1} = 1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + ...... + \infty$$

Then average of these two function will give,

$$\frac{e + e^{-1}}{2} = 1 + \frac{1}{2!} + \frac{1}{4!} + .... + \infty$$

**DE-MOIVER's Theorem:**

If n is integer positive or negative then,

$$(Cos\theta + iSin\theta)^n$$

And if n is a fraction it may be positive or negative then the $Cosn\theta + iSinn\theta$ is one of the value of $(Cos\theta + iSin\theta)^n$ .

**Proof:**

The proof of the DE-MOIVER's theorem can be explained in three cases.

**Case-I:**

Here n is taken as positive integer. So we get,

$$(Cos\alpha + iSin\alpha)(Cos\beta + iSin\beta)$$

$$= Cos\alpha Cos\beta - Sin\alpha Sin\beta + i(Sin\alpha Cos\beta + Cos\alpha Sin\beta)$$

$$= Cos(\alpha + \beta) + iSin(\alpha + \beta)$$

Similarly multiplying both side by $(Cos\gamma + iSin\gamma)$ we get,

$$(Cos\alpha + iSin\alpha)(Cos\beta + iSin\beta)(Cos\gamma + iSin\gamma)$$

$$= Cos(\alpha + \beta + \gamma) + iSin(\alpha + \beta + \gamma)$$

$(Cos\alpha + iSin\alpha)(Cos\beta + iSin\beta)(Cos\gamma + iSin\gamma)$....to n factors

$Cos(\alpha+\beta+\gamma+.....n-terms)+iSin(\alpha+\beta+\gamma+.....n-terms)$

If $\alpha=\beta=\gamma=\theta$ then the above equation will become,

$(Cos\theta + iSin\theta)^n = (Cosn\theta + iSinn\theta)$

So this how the DE-MOIVER's theorem can be proved if n is taken a positive integer. Similarly the same theorem can be prove for the negative integer $n=-m$.

$(Cos\theta + iSin\theta)^n = (Cos\theta + iSin\theta)^{-m}$

$= \frac{1}{(Cos\theta + iSin\theta)^m} = \frac{1}{(Cosm\theta + iSinm\theta)}$

$= \frac{Cosm\theta - iSinm\theta}{(Cosm\theta + iSinm\theta)(Cosm\theta - iSinm\theta)}$

$= Cosm\theta - iSinm\theta$

$= Cos(-m\theta) + iSin(-m\theta)$

$= Cosn\theta + iSinn\theta$

So this is how the DE-MOIVER's theorem can be proved for the n as negative integer. Similarly these can be proved for the fraction as,

$n = \frac{p}{q}$

$\left( Cos\frac{P\theta}{q} + iSin\frac{P\theta}{q} \right)$

This is one of the value of

$(Cos\theta + iSin\theta)^{\frac{p}{q}}$

$(Cos\theta + iSin\theta)^n$

Here we can prove that the DE-MOIVER's theorem is applicable to fraction also. So using this theorem we can solve this example.

**Example:**

Using DE-MOIVER's theorem prove that $\left(\frac{1+Cos\phi+iSin\phi}{1+Cos\phi-iSin\phi}\right)=Cosn\phi+iSinn\phi$ .

**Solution:**

Let us take the L.H.S of the equation and substituting certain trigonometric function we get,

$$=\left\{\frac{2Cos^2\frac{\phi}{2}+2iSin\frac{\phi}{2}Cos\frac{\phi}{2}}{2Cos^2\frac{\phi}{2}-2iSin\frac{\phi}{2}Cos\frac{\phi}{2}}\right\}$$

$$=\left\{\frac{Cos\frac{\phi}{2}+iSin\frac{\phi}{2}}{Cos\frac{\phi}{2}-iSin\frac{\phi}{2}}\right\}^n$$

Substituting the DE-MOIVER's theorem this expression will become,

$$=\left\{\left\{Cos\frac{\phi}{2}+iSin\frac{\phi}{2}\right\}^2\right\}^n=Cosn\phi+iSinn\phi$$

**Summary:**

In this lecture we learned that the Exponential function or algebraic function will be expanded as series.

$$e^x=\left[1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+......+\infty\right]$$

Then the same can be applied for the complex function also. Then we learned the DE-MOIVER's theorem gives the power of the complex exponential function.

After listening to this lecture you can answer the following questions.

**Questions:**

1. Define exponential function.

2. State DE-MOIVER’s theorem.
3. Expand $e^{(i2x)}$.