

FAQs

1. Find the value of e^{-1} .

Solution:

According to the expansion of exponential function

$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$. Now substitute -1 in the place of x, so we get

$$e^{-1} = 1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \dots$$

$$= 1 - \frac{1}{1} + \frac{1}{1 \times 2} - \frac{1}{3 \times 2 \times 1} + \dots$$

$$= \frac{1}{2} - \frac{1}{6} \text{ (neglecting higher factorials)}$$

$$= \frac{3-1}{6} = \frac{2}{6} = \frac{1}{3}$$

Approximately $e^{-1} = \frac{1}{3}$

2. What is the expansion of e^{ax} ?

Solution:

$$e^{ax} = 1 + \frac{ax}{1!} + \frac{(ax)^2}{2!} + \frac{(ax)^3}{3!} + \dots \infty$$

$$= \sum_{n=0}^{\infty} \frac{(ax)^n}{n!}$$

3. Find the sum of the series $\sum_{n=0}^{\infty} \frac{n^2 - n + 1}{n!}$ is

Solution:

$$T_n = \frac{n^2 - n + 1}{n!} = \frac{n(n-1)}{n!} + \frac{1}{n!}$$

$$= \frac{1}{(n-2)!} + \frac{1}{n!}$$

$$\therefore \sum_{n=0}^{\infty} T_n = \sum_{n=0}^{\infty} \frac{1}{(n-2)!} + \sum_{n=0}^{\infty} \frac{1}{n!}$$

$$= e + e = 2e$$

4. State De Moivre's Theorem.

Solution:

De Moivre's theorem states that if 'n' is any integer, positive or negative then $(\cos \theta + i \sin \theta)^n = \cos n\theta + i \sin n\theta$ and if 'n' is a fraction positive or negative then $\cos n\theta + i \sin n\theta$ is one of the value of $(\cos \theta + i \sin \theta)^n$.

5. Find the co-efficient of x^n in the expansion of e^{a+bx}

Solution:

$$e^{a+bx} = e^a \cdot e^{bx}$$

$$= e^a \left[1 + \frac{bx}{1!} + \frac{(bx)^2}{2!} + \dots + \frac{(bx)^n}{n!} + \dots \right]$$

So co-efficient of x^n in this expansion is

$$= \frac{e^a \cdot b^n}{n!}$$