Bachelor of Architecture

Mathematics

Lecture 16

In this lecture we are going to see the Probability, the Laws of addition and multiplication of probability, Conditional probability, Independent events and then finally summary.

Probability:

Probability is a concept which numerically measure the degree of uncertainty and therefore of certainty of the occurrence of events. If an events A can happen in 'm' mays and fail in 'n' ways being equally likely to occur then the probability of the happening of A is given as,

$$= \frac{no.of\,.\,favourable.cases}{total.no.of\,.mutually.exclusive\,\&\,equally.like.cases}$$

$$= \frac{m}{m+n}$$

And that of its failing is defined as,

$$= \frac{m}{m+n}$$

If the probability of the happening is p and the probability of not happening is q. Then

$$= \frac{m}{m+n} + \frac{n}{m+n} = \frac{m+n}{m+n} = 1.or.p+q=1$$

Equally likely events:

Two events are said to be equally likely if one of them cannot be expected in preference to other. For instance pack, we may get any card then the 52 different cases are equally likely. This is how the equally like events can be defined. The next thing is Independent events.

Independent Events:

Two events may be independent when the actual happening of one does not influence in any way the probability of the happening of the other. For example the event of getting head on first coin and the event of getting tail on the second coin in a simultaneous throw of two coins are independent. So that two events are said to be independent.

Mutually exclusive events:

Two events are known as mutually exclusive when the occurrence of one of them excludes the occurrence of the other. For example on tossing of a coin either we get head or tail but not both. Now we will understand this concept by doing a problem.

Example 1:

Find the probability of getting a) 5 and b) an even number with an ordinary six faced die.

Solution:

For the first case we are asked to find getting 5. So it will be given by,

$$= \frac{no.of.fa\mathrm{var}\,ble}{total.no.of.events}$$

$$= \frac{1}{6}$$

So this is the solution for the case of getting 5. Then the second case is no of ways get even number will be,

$$=\frac{3}{6}$$

$$=\frac{1}{2}$$

So this is the solution for the second case.

Example 2:

Find the probability of getting 9 with two dice.

Solution:

Here two dice were thrownand we are asked to find the probability of getting 9. For that we need sum the occurrence of the two ways. So the possibilities will be,

No of ways to get 9 are $(3+6),(4+5),(5+4),(6+3)$. That is the required probability is,

$$=\frac{4}{36}=\frac{1}{9}$$

Example 3:

From a pack of 52 cards one is drawn at random. Find the probability of getting a king.

Solution:

The solution to this problem will be obtained by taking the no of king. And the total no of cards is 52. So a king can be chosen in four ways. So the probability will be,

$$=\frac{4}{52}=\frac{1}{13}$$

Addition law of probability:

If $p_1, p_2, \ldots\ldots. p_n$ be separate probability of mutually exclusive events. Then the probability p that any of these events will happen is given by,

$$p = p_1 + p_2 + p_3 +, \ldots\ldots. p_n$$

This is also called as the theorem of addition of probability.

Proof:

Let A,B,C be the events happening then the probability of this events will be

p_1, p_2, p_3. Now 'n' be the total number of favorable cases. Then $m_1 + m_2 + m_3 + \ldots\ldots + m_n$ will be the total no of favorable cases for the event either A or B or C to be occur.

Hence the probability of $p(A + B + C + \ldots\ldots)$ will be

$$= \frac{m_1 + m_2 + m_3 + \ldots\ldots + m_n}{n}$$

$$= \frac{m_1}{n} + \frac{m_2}{n} + \frac{m_3}{n} + \ldots\ldots + \frac{m_n}{n}$$

$$= p(A) + p(B) + p(C) + \ldots\ldots p(n)$$

So this is how the addition law of probability can be proved.

Multiplication law of probability:

If there are two independent events the respective probability of which are known then the probability that both will happen is the product of the probability of their happening respectively,

$$p(AB) = p(A) + p(B)$$

So this is the multiplication law of probability and this can be proved as follows.

Proof:

Suppose A and B are the two independent events then the event A happens in m_1 ways and the failing of A will be denoted as n_1. So the probability for A is,

$$p(A) = \frac{m_1}{m_1 + n_1}$$

In a similar manner the event B happens in m_2 ways and the failing of B will be denoted as n_2. So the probability for B is,

$$p(B) = \frac{m_2}{m_2 + n_2}$$

Therefore there are four possibilities that is A and B both may happen as $m_1.m_2$. Then A may happen and B may fail that is $m_1.n_2$. The third case is A may fail and B may happen as $n_1.m_2$. Now the four cases is both may fail and we get the probability as,

$$p(A).p(B)$$

Condition probability:

The probability of happening an event A such that event B has already happened is called the conditional probability of happening of A on the condition that B has already happen it is usually denoted by $p(\frac{A}{B})$. Now we can apply these concepts of probability in a problem.

Example 4:

A bag contains four white and two black balls and second bag contains three of each color. A bag is selected at random and a ball is then drawn at random from the bag chosen. What is the probability that the ball drawn is white?

Solution:

If the first bag is chosen then its probability will be $\frac{1}{2}$. Then the probability of choosing white balls is $\frac{4}{6}$. Therefore the probability of drawing white ball from the bag is,

$$\frac{1}{2} \times \frac{4}{6} = \frac{1}{3}$$

Then the same white ball is chosen from the second ball will be,

$$\frac{1}{2} \times \frac{3}{6} = \frac{1}{4}$$

These two steps are mutually exclusive so for this the addition law of probability can be used. Then the total probability will be

$$\frac{1}{3} + \frac{1}{4} = \frac{7}{12}$$

Summary:

In this lecture we have learnt that the probability measures the degree of uncertainty and certainty of the occurrence of events. Addition law of probability gives $p = p_1 + p_2 + p_3 +, ...p_n$. Then the multiplication law of probability give $p(AB) = p(A) + p(B)$. Condition probability and independent events are also discussed. So after listening to this lecture you can answer the following questions.

Questions:

1. Define probability of a event.
2. Give the addition and multiplication law of probability.
3. What is the probability that a leap year selected at random will contain 53 Sundays?