Bachelor of Architecture

Mathematics

Lecture 1

In this lecture we are going to discuss the Trigonometric functions in that Sine function, Cos function, Tan function and then finally summary.

Sine Function

We know sine function is a trigonometric function which can be obtained from the right angle triangle. Consider a right angle triangle in that one angle will be 90 degree and sum of there angles will also be 90 degree. Then sine opposite to that is considered as opposite sine then sine opposite to right angle is assumed as hypotenuse, then sine adjacent to the angle is the adjacent angle.

$$Sin\theta = \frac{opposite}{hypotenuse}$$

Apart from this sine function can also be obtained by assuming aunit circle. Here a unit circle is assumed center of that circle is fixed at the origin of the co-ordinate system. We know unit circle means a circle with radius magnitude should be one.Then a line is drawn from the center of the circle to any point on the circumference of the circle. Now the value of this can be x,y. Since it is a unit circle the value of y function is one and it is denoted as $Sinx$. This is how the sine function can be defined by assuming a unit circle.Now this sin function is a periodic function we know the periodic function is the function which repeats after certain meter. That is, $Sin(0) = 0$ and $Sin(90) = 1$

Like that the values for x and sine function is given in the tabular column,

X	0^0	45^0	90^0	135^0	180^0	225^0	270^0	315^0	360^0
Sinx	0	0.71	1	0.71	0	-0.71	-1	-0.71	0

So here the function has reached the maximum positive value at 90 degree and negative maximum value at 370 degree. When we increase the x value further then the value of sin x will again reach zero. So we get a wave function. When we repeat this further again the value change negative and the positive to give a complete wave function. Here we can say the sine function is a periodic function. In this function instead of degree if we you the radian again will get the same pattern.So far we seen the value of the function for the positive side if we increase the value in the negative side of the sine function we get the same periodic function.

The domine of the sine function will be 1 and -1. So sine function is the function with period 2π radian or 360 degree.

Periodicity of sine function:

As we said earlier the sine function repeat itself every 360 degree or 2π radians. So that in equation the sine function can be written as,

$$Sinx = Sin(x + 2\pi)$$

Cos function

To define the Cos function again the same right angel triangle is taken, here the angle is assumed in these right angle triangle and the definition is given by,

$$Cos\theta = \frac{adjacent}{hypotenouse}$$

Now we imagine a unit circle to understand the Cos function. Center of that unit circle is fixed at the origin of the co-ordinate system. Then connect to any point in the circumference of the circle with the origin of the co-ordinate system. Then angle subtended by the line with the horizontal line is considered as x. the value of trigonometric function is defined by the x-axis.

These Cos function is also a periodic function. To understand these periodic function we need to substitute values of x in the Cos function.

X	0^0	45^0	90^0	135^0	180^0	225^0	270^0	315^0	360^0
Cos x	1	0.71	0	-0.71	-1	-0.71	0	0.71	1

Here if we substitute the values that Cos 0 degree is 1 and Cos 90 degree is 0, the value of Cos 45 degree will be somewhere between 1 and 0 that is 0.71. Then again the value of Cos 360 degree will be 1. So this is how we can say that the Cos function is the periodic function. The domine of the Cos function will be -1 to 1.

Now we will compare the Cos function with the Sine function. Here Cos x is 1 for the x value 0 but in the sine function the value is 1 for the x value 0 degree. So there is a phase difference between the sine function and the Cos function.And the phase difference between this functions is given by $\frac{\pi}{2}$.

Periodicity of Cos function:

These Cos function also repeat the function for every 360 degree hence this Cos function is periodic function with 360 degree.The Cos function is periodic in both domine and the periodicity of this Cos function is 2π .

$$f(x) = Cosx = Cos(x + 2\pi)$$

$$Cosx = Cos(x + 2n\pi)$$

This is how the periodicity of the Cos function can be learned.

Tan function

Tan function in a right angle triangle can be define as,

$$\tan\theta = \frac{opposite}{adjacent}$$

Here will see the values of the tan function along with the Sine and Cos values also,

X	0^0	45^0	90^0	135^0	180^0	225^0	270^0	315^0	360^0
Sin x	0	0.71	1	0.71	0	-0.71	-1	-0.71	0
Cos x	1	0.71	0	-0.71	-1	-0.71	0	0.71	1
tan x	0	1	*	-1	0	1	*	-1	0

The Sin function and the Cos function is given here because the tan function can be obtained by taking the values of Sine and Cos function. We know the basic trigonometric formula that,

$$\tan\theta = \frac{Sin\theta}{Cos\theta}$$

So the value of tan function is given by the ratio of the sin function and the Cos function.As we seen earlier the tan function depends on the sine function and the Cos function. The sine function and the Cos function both are a periodic function hence the tan function is also a periodic function. But the function in sine and Cos has the long interval and the tan function has the short interval so the periodicity of this tan function is low compared to the periodicity of the sine function and the Cos function.

Periodicity of tan function:

As tan depends on sine and Cos function it repeat itself every 180 degree or for π radians. So this can be written in equation as,

$$f(x) = \tan x = \tan(x + \pi)$$

$= \tan(x + n\pi)$

So this is how the periodicity of tan function can be defined and explained. From this discussion we have learned the sine function, Cos function and the tan function are all periodic function which can be used in many fields.

Now we will see a problem which uses the trigonometric functions.

Example 1:

Find the height of the chimney when it is found that on waking towards it 100 meters in a horizontal line through its base the angular elevation of its top changes from 30^0 to 45^0.

Solution:

Let consider the top point of the chimney as P and the base point as O. Then the height of the chimney is represented by the line OP. In this problem we are asked to find the height of the chimney, so let us keep the height of the chimney as h.

Then a man is waking towards the chimney in a horizontal line through its base. The angular elevation of the top of the chimney will be changes from 30 to 45 degrees. The point OA will be $100 + h$.

$\angle PAO = 30^0$

$\angle PBO = 45^0$

$AB = 100 meters$

In a right angle triangle ΔPOB,

$\angle BPO = \angle PBO = 45^0$

$OB = OP = h$

Then we know that,

$OA = OB + AB = h + 100$

In a right angle triangle ΔPOA,

$\tan 30^0 = \frac{OP}{OA}$

$$\frac{1}{\sqrt{3}} = \frac{h}{h+100}$$

$$\sqrt{3}h = h + 100$$

$$h(\sqrt{3} - 1) = 100$$

$$h = 50(\sqrt{3} + 1) meters$$

Summary:

So from this lecture we have learned that the Sine function is a periodic function of periodicity 2π. And the Cos function is also a periodic function of periodicity 2π. Hence Tan function depends only on sine and Cos function it is also a periodic function of periodicity π.

After listening to this lecture you can answer the following questions.

Questions:

1. Define sine function.
2. Give the pictorial representation of Cos function.
3. If $A + B = 45^0$ prove that $(1 + \tan A)(1 + \tan B) = 2$.